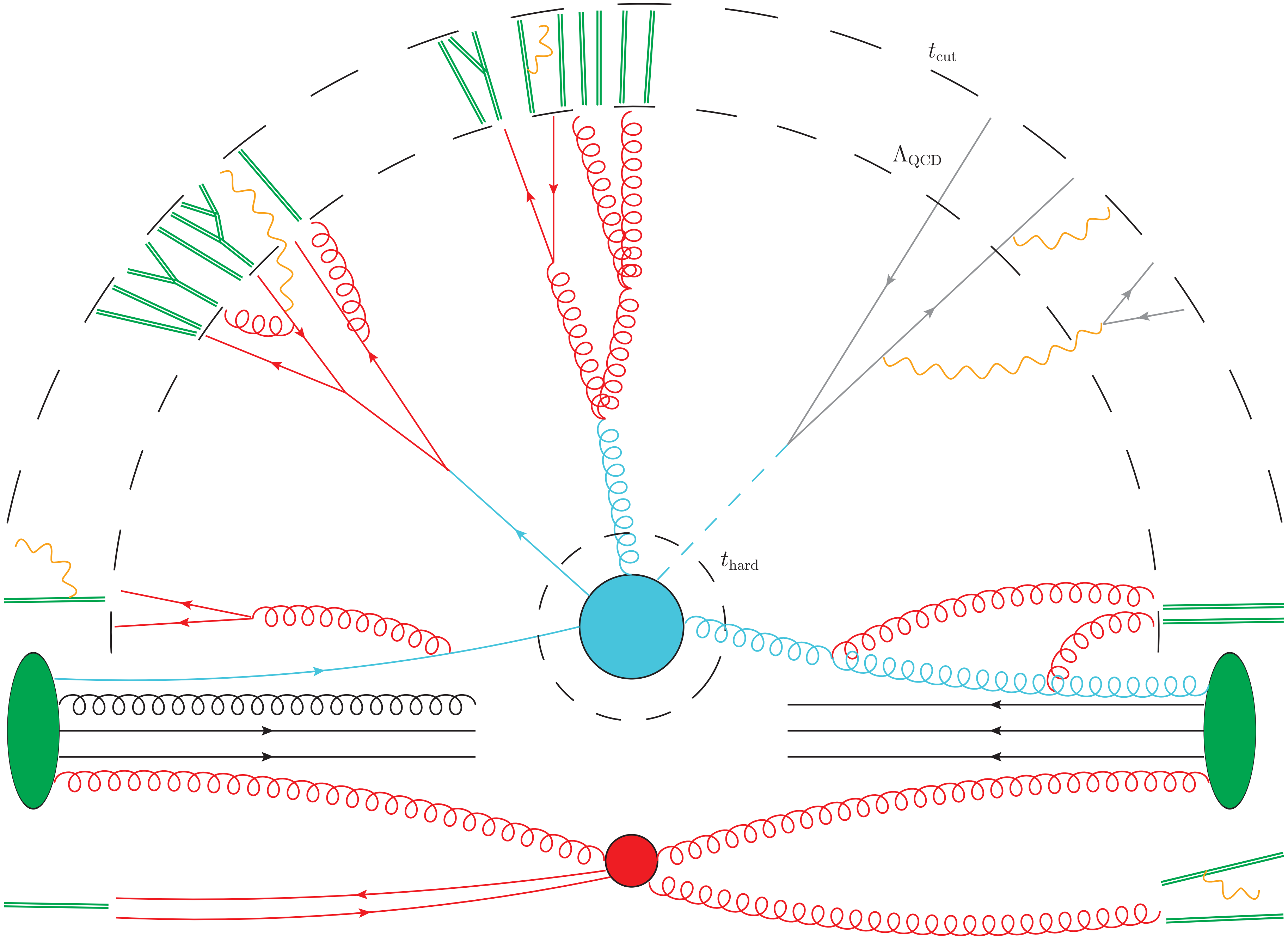
Electroweak Radiation in Antenna Showers

Rob Verheyen

With Ronald Kleiss

Introduction



Introduction

Parton Showers = Resummation

Photon Emission

Soft and collinear logarithms

Current implementations: only collinear

Photon Splitting

Only collinear logarithms

Cast in antenna formalism

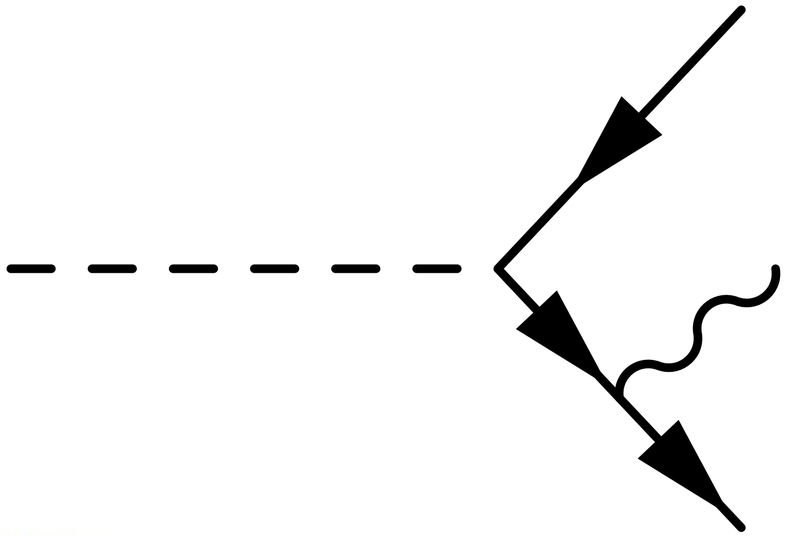
Electroweak Radiation

Complications due to mass and spin

Follow QCD antenna shower Vincia

Giele, Kosower, Skands:1102.2126

Gehrmann, Ritzmann, Skands:1108.6172



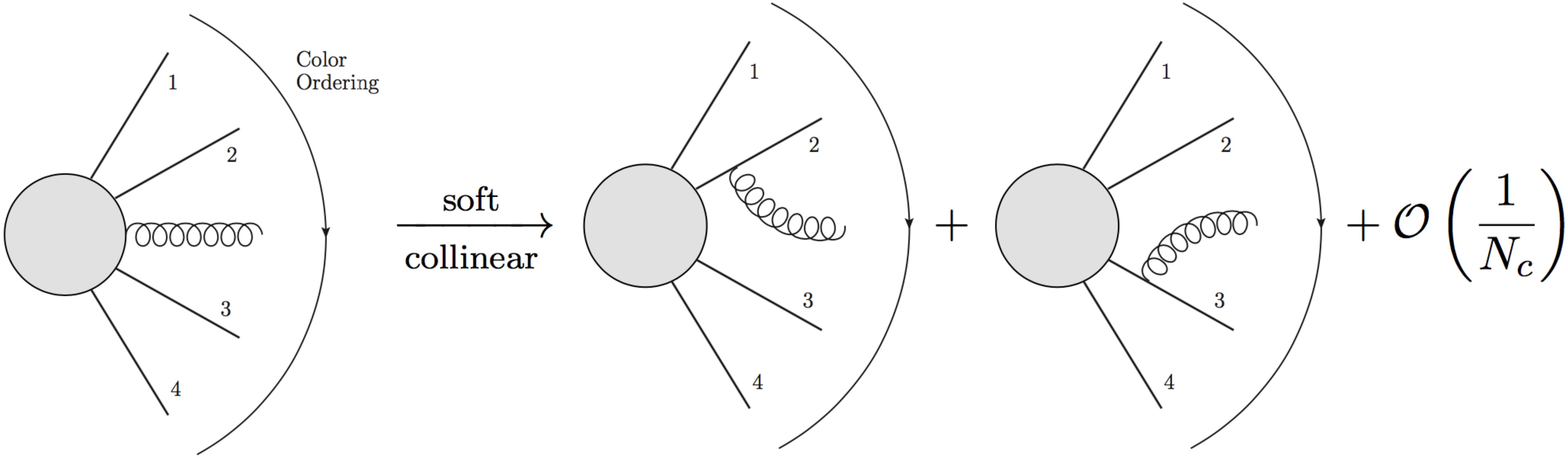
Photon Emission

Leading Color Gluon Emission

Factorization

$$|M(..,p_a,k,..)|^2 \xrightarrow{p_a \| k} g^2 C \frac{P(z)}{p_a \cdot k} |M(..,p_a+k,..)|^2$$

$$|M(..,p_a,k,p_b,..)|^2 \xrightarrow{k \to 0} g^2 C \left[\frac{2p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} - \frac{m_a^2}{(p_a \cdot k)^2} - \frac{m_b^2}{(p_b \cdot k)^2} \right] |M(..,p_a,p_b,..)|^2$$



Leading Color Gluon Emission

Factorization

$$|M(..,p_a,k,..)|^2 \xrightarrow{p_a\|k} g^2 C\,\frac{P(z)}{p_a\cdot k}\,|M(..,p_a+k,..)|^2$$

$$|M(..,p_a,k,p_b,..)|^2 \xrightarrow{k\to 0} g^2 C\left[\frac{2p_a\cdot p_b}{(p_a\cdot k)(k\cdot p_b)} - \frac{m_a^2}{(p_a\cdot k)^2} - \frac{m_b^2}{(p_b\cdot k)^2}\right]|M(..,p_a,p_b,..)|^2$$

$$|M(..,p_a,k,p_b,..)|^2 \approx g^2 C\, a_e^{QCD}(p_a,k,p_b)|M(..,p'_a,p'_b,..)|^2$$

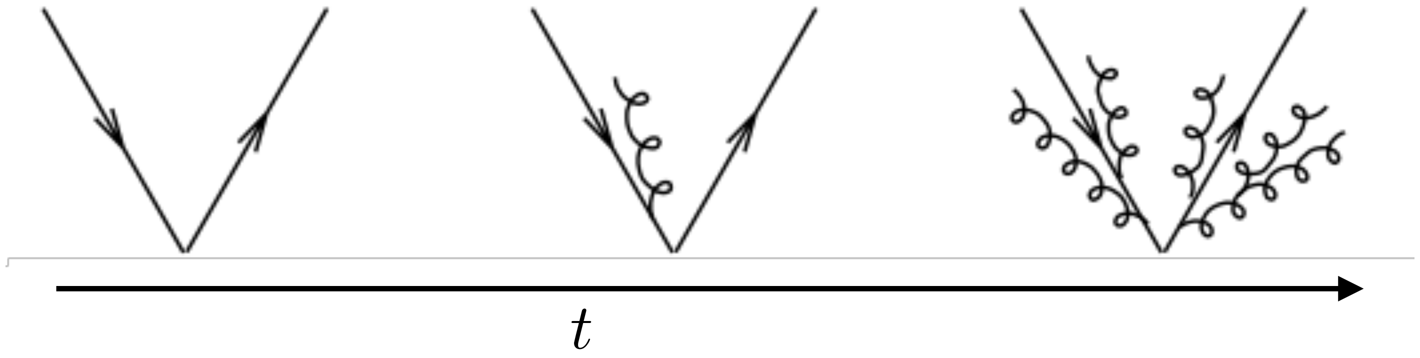
$2 \to 3$ branching

Computing antennae

$$a_e^{QCD} = \frac{|M(X\to p_a,k,p_b)|^2}{|M(X\to p'_a,p'_b)|^2}$$

Gluon Emission Ordering

Ordering scale $t = 4p_\perp^2 = 16\frac{(p_a \cdot k)(p_b \cdot k)}{m^2}$



Cutoff on t $\rightarrow$ removes singular regions

Strong ordering $t_1 > t_2,\ t_2 > t_3$ etc..

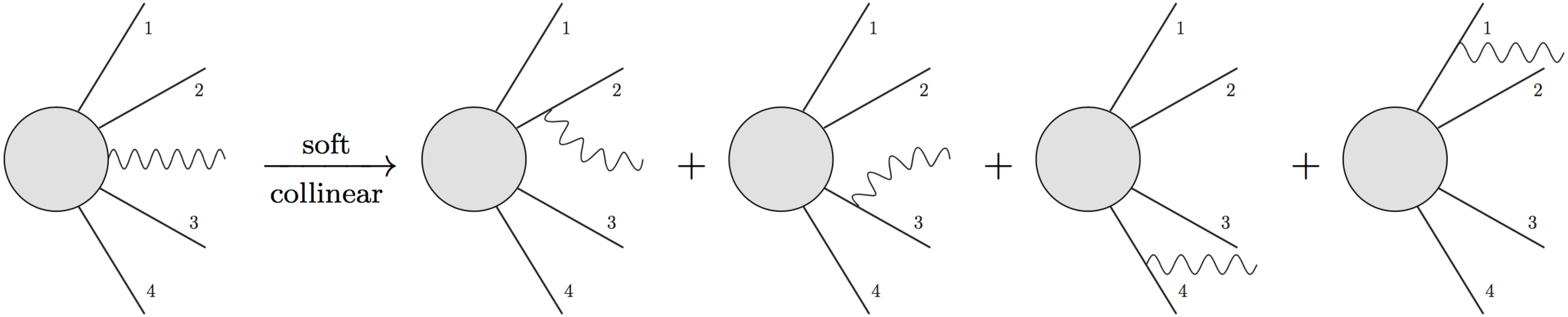
Illustration: S. Galam

Photon Emission

Factorization

$$|M(..,p_a,k,..)|^2 \xrightarrow{p_a\|k} e^2 Q_a^2 \frac{P(z)}{p_a\cdot k}|M(..,p_a+k,..)|^2$$

$$|M(\{p\},k)|^2 \xrightarrow{k\to 0} -e^2 \sum_{[a,b]} Q_a Q_b \left[\frac{2p_a\cdot p_b}{(p_a\cdot k)(k\cdot p_b)} - \frac{m_a^2}{(p_a\cdot k)^2} - \frac{m_b^2}{(p_b\cdot k)^2}\right]|M(\{p\})|^2$$



Photon Emission

Factorization

$$|M(.., p_a, k, ..)|^2 \xrightarrow{p_a \| k} e^2 Q_a^2 \frac{P(z)}{p_a \cdot k} |M(.., p_a + k, ..)|^2$$

$$|M(\{p\}, k)|^2 \xrightarrow{k \to 0} -e^2 \sum_{[a,b]} Q_a Q_b \left[\frac{2 p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} - \frac{m_a^2}{(p_a \cdot k)^2} - \frac{m_b^2}{(p_b \cdot k)^2} \right] |M(\{p\})|^2$$

$$|M(\{p\}, k)|^2 \approx e^2 a_e^{QED}(\{p\}, k)\, |M(\{p'\})|^2$$

$$a_e^{QED}(\{p\}, k) = -\sum_{[a,b]} Q_a Q_b \left[2 \frac{p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} - \frac{m_a^2}{(p_a \cdot k)^2} - \frac{m_b^2}{(p_b \cdot k)^2} + \frac{1}{m_{abk}^2 - m_a^2 - m_b^2} \left(\frac{p_a \cdot k}{p_b \cdot k} + \frac{p_b \cdot k}{p_a \cdot k} \right) \right]$$

$n \to n + 1$ branching

Radboud University

Photon Emission Ordering

Separate phase space into *sectors*

$2 \to 3$ branching

$$|M(\{p\},k)|^2 \approx \sum_{[a,b]} a_e(\{p\},k)\,\theta((p_\perp^2)_{ab})\,|M(..,p'_a,p'_b,..)|^2$$

1 if $(p_\perp^2)_{ab}$ is the smallest

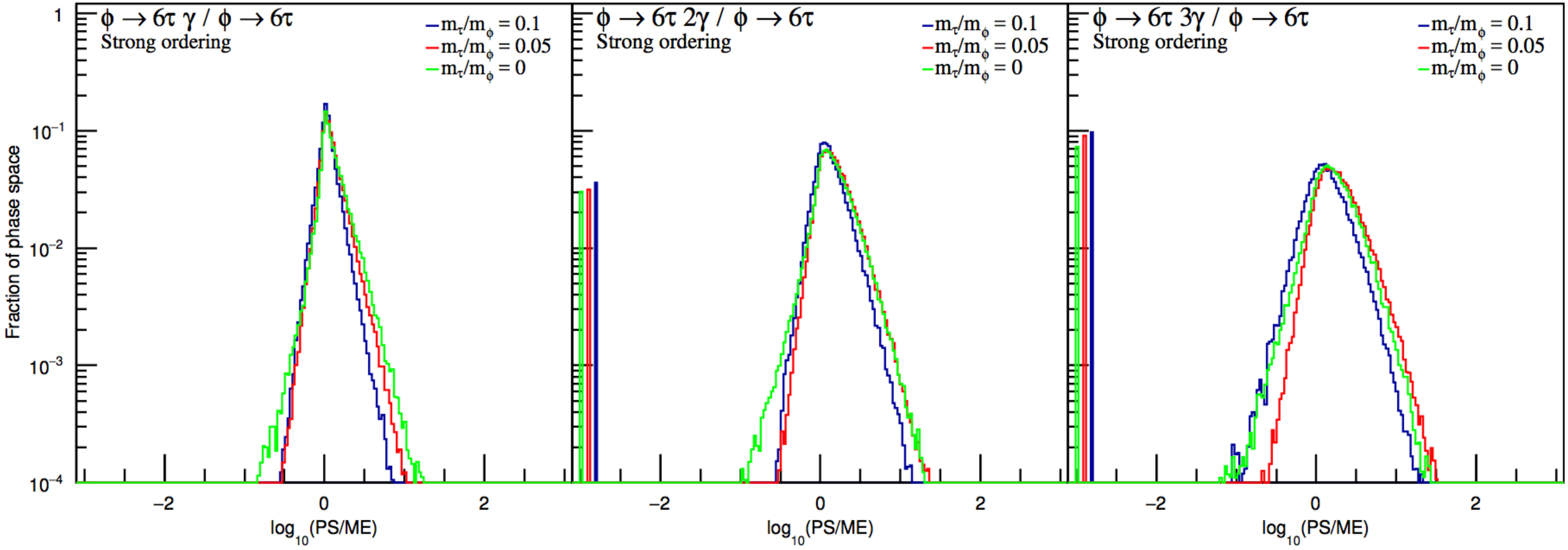
Equivalent to ordering in

$$t = 4\min\left((p_\perp^2)_{ab}\right) = 16\min\left(\frac{(p_a\!\cdot\! k)(p_b\!\cdot\! k)}{m^2}\right)$$

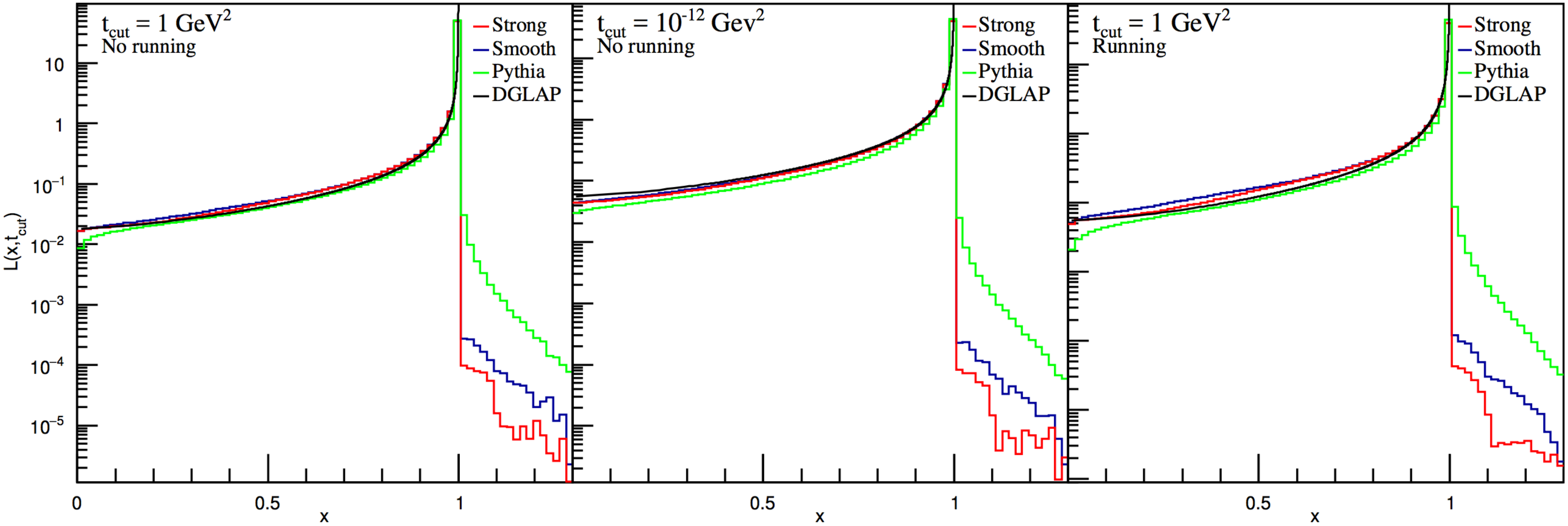
Matrix Element Comparison

- Sample phase space uniformly using RAMBO
- Compute matrix elements with Madgraph

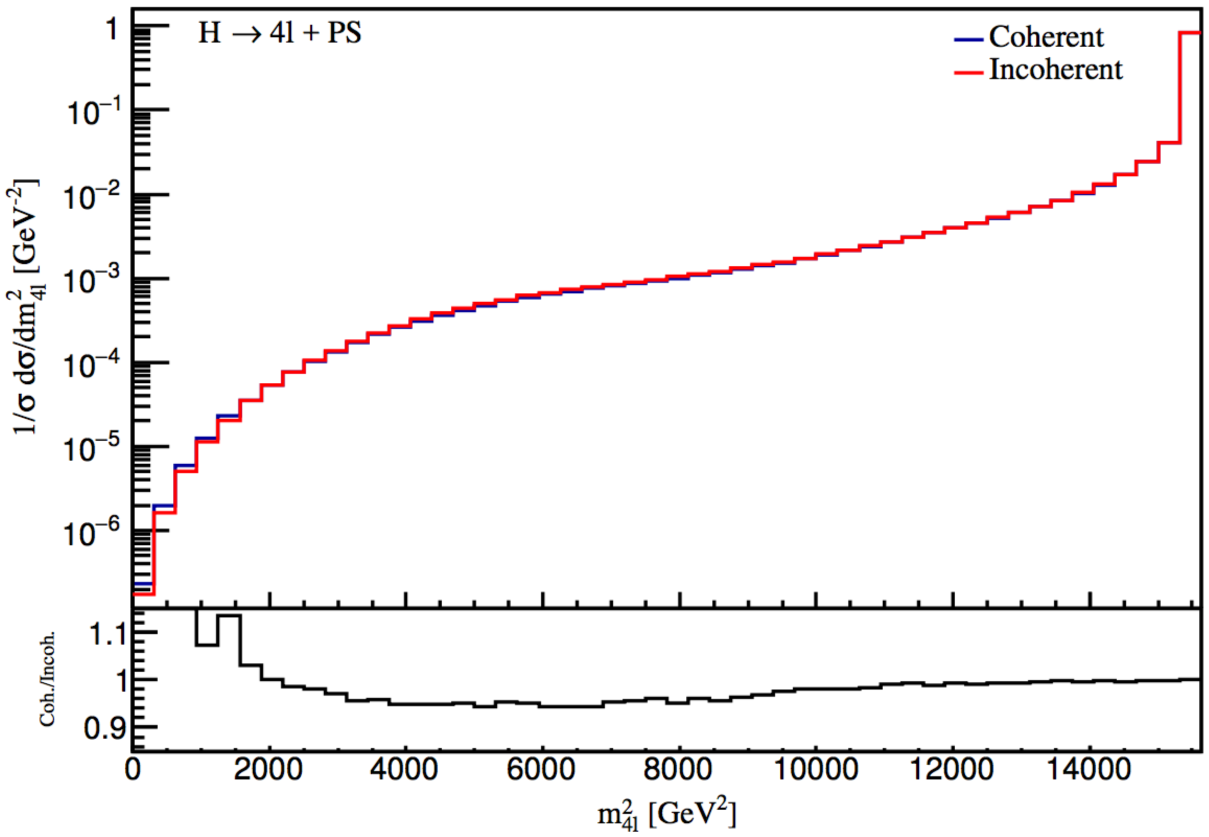
$$\frac{PS}{ME} = \frac{\sum_{histories} a_1 \ldots a_{n-m} |M_m|^2}{|M_n|^2}$$

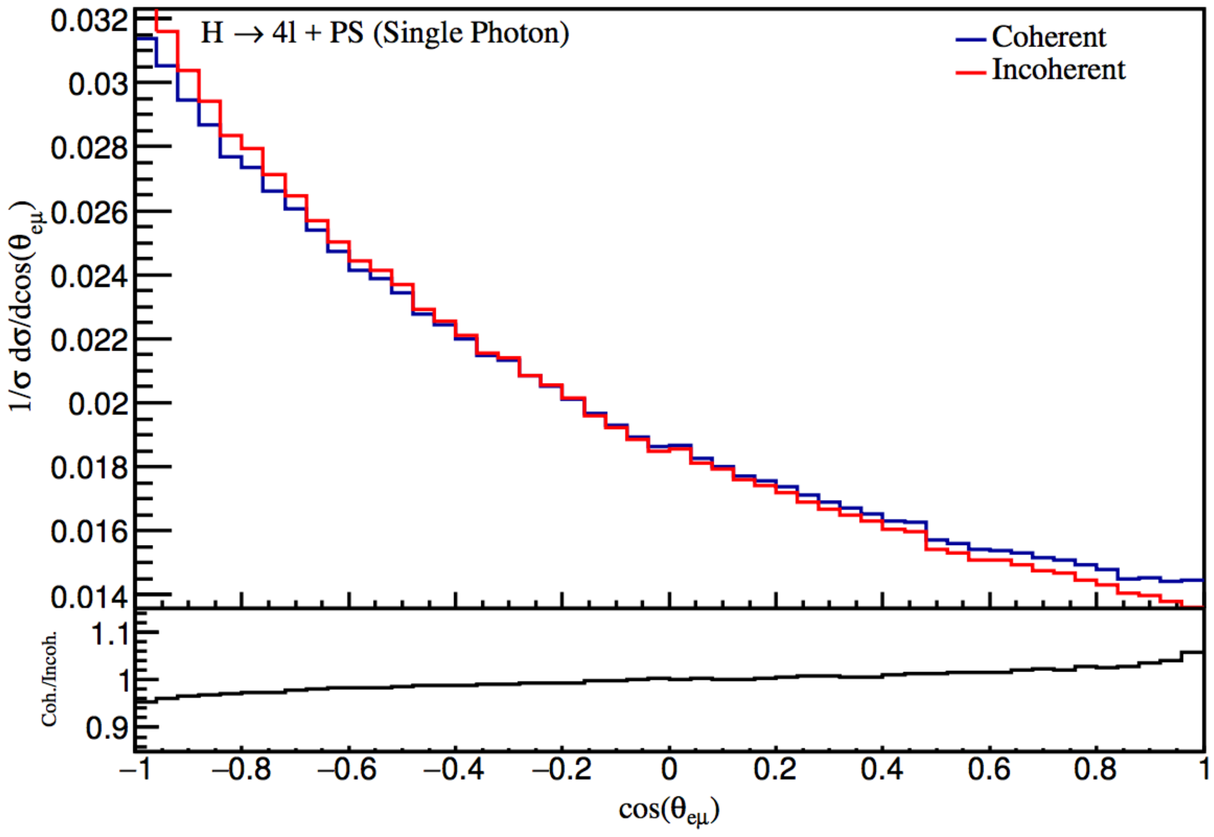


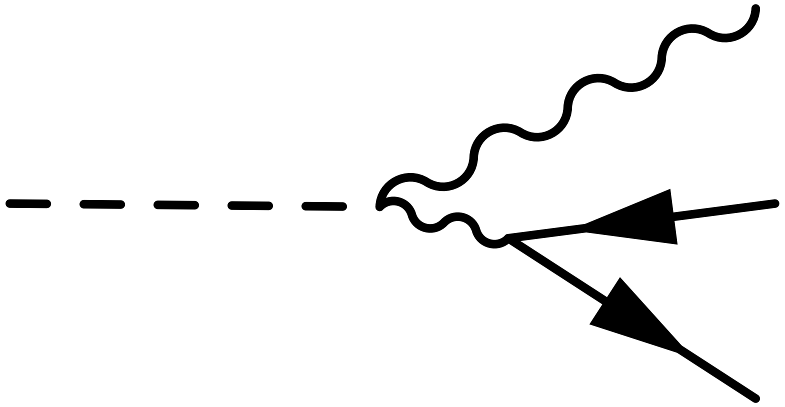
Comparison - DGLAP equation



Comparison - Coherence







Photon Splitting

Photon Splitting

Factorization

$$\boxed{|M(..,p_a,p_b)|^2 \xrightarrow{p_a \| p_b} e^2 Q_f^2 \frac{P_s(z)}{p_a{\cdot}p_b + m_f^2} |M(..,k)|^2}$$

→

$$\begin{aligned} t &= m_{ab}^2 \\ &= 2(p_a{\cdot}p_b + m_f^2) \end{aligned}$$

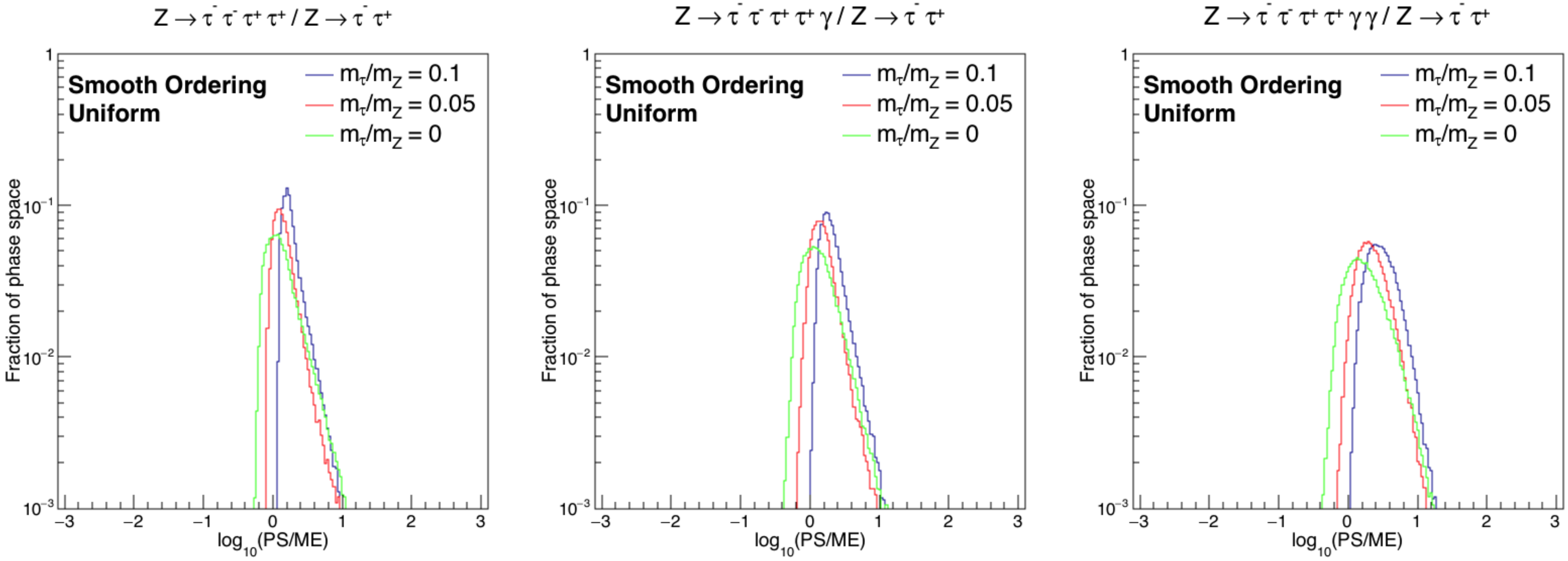
Antenna showering $\longrightarrow$ requires *spectator*

$$a_s^{QED}(p_a,p_b,q) = \frac{Q_f^2}{p_a{\cdot}p_b + m_f^2}\left[4\frac{(p_a{\cdot}q)^2 + (p_b{\cdot}q)^2}{m_{abq}^2} + \frac{m_f^2}{p_a{\cdot}p_b + m_f^2}\right]$$

In QCD: Choice of spectator limited by color ordering
In QED: Anything goes

Selecting the Spectator

First attempt: Select spectator uniformly



What's causing this overcounting?

Ariadne factor

Emission $\rightarrow p_K$ is on-shell

Splitting $\rightarrow p_K$ is taken off-shell

Giele, Kosower, Skands:1102.2126

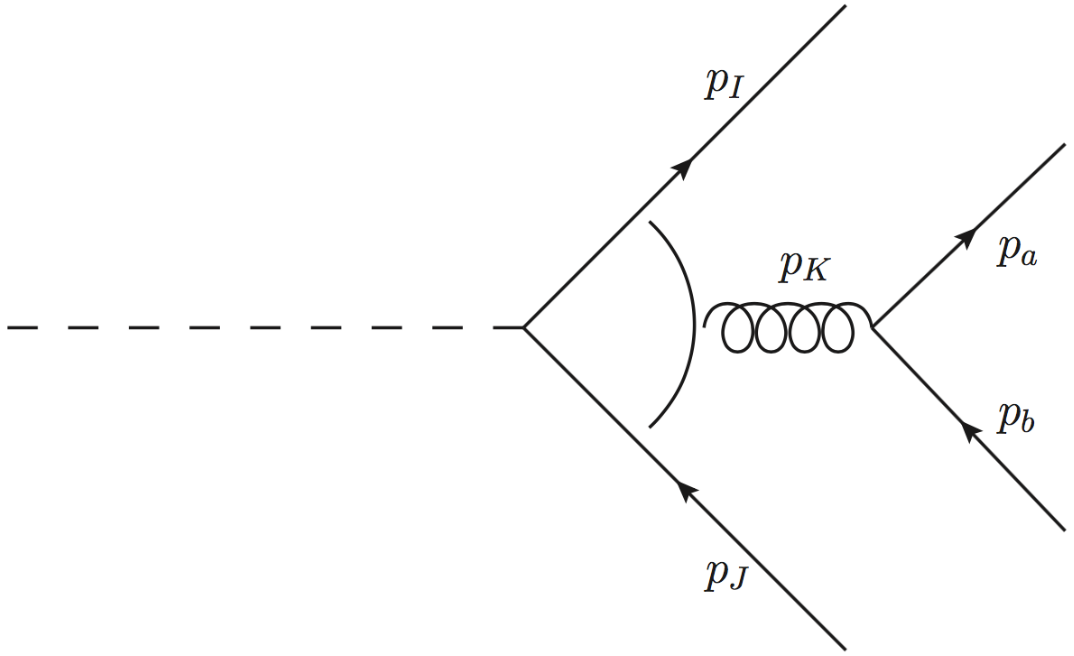
Lönnblad: Comput.Phys.Commun. 71 (1992) 15-31

Let’s say p_K is collinear with p_I $\rightarrow m^2_{IK} = (p_I + p_K)^2$ is small

- Use p_I as spectator $\rightarrow m^2_{IK}$ stays the same
- Use p_J as spectator $\rightarrow m^2_{IK}$ becomes large

$$p^{\mathrm{Ari}}_{IK} = \frac{m^2_{JK}}{m^2_{IK} + m^2_{JK}}$$

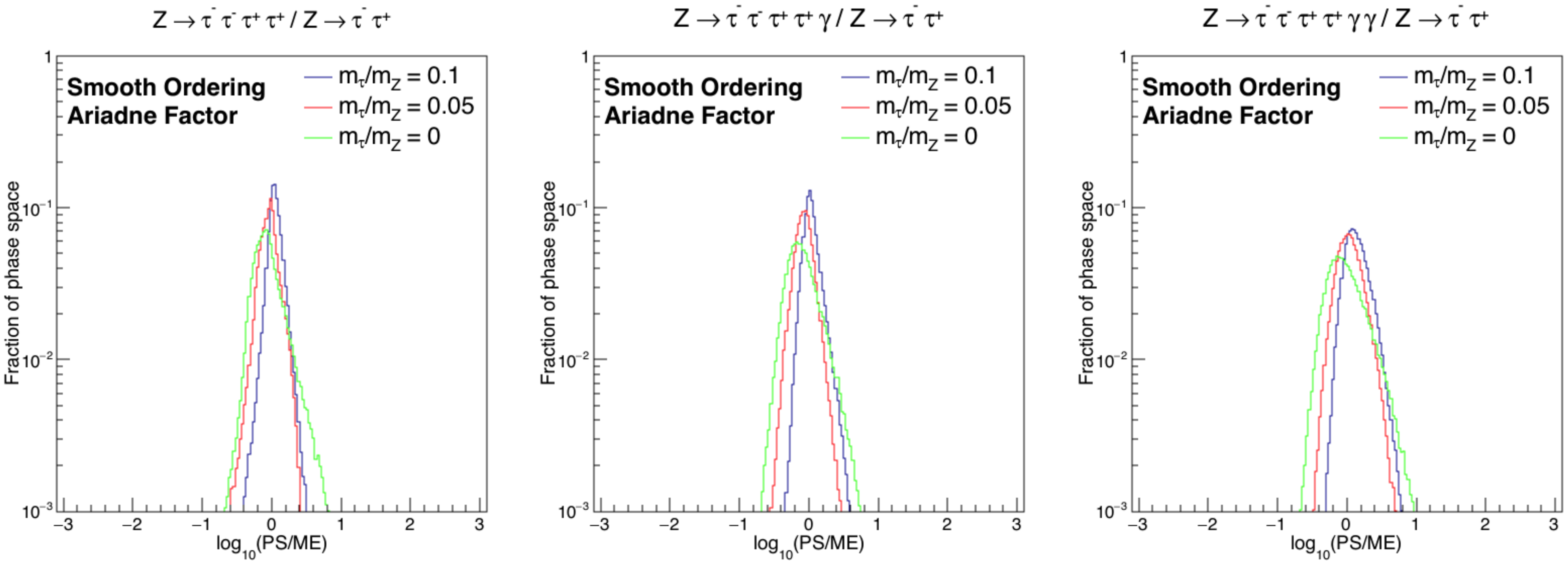
Probability to select p_I as spectator



Selecting the Spectator

Generalized Ariadne factor

$$p_{IK}^{\mathrm{Ari}} = \frac{1/m_{IK}^2}{\sum_J 1/m_{JK}^2}$$



Electroweak Radiation

Work in progress

Importance of EW radiation

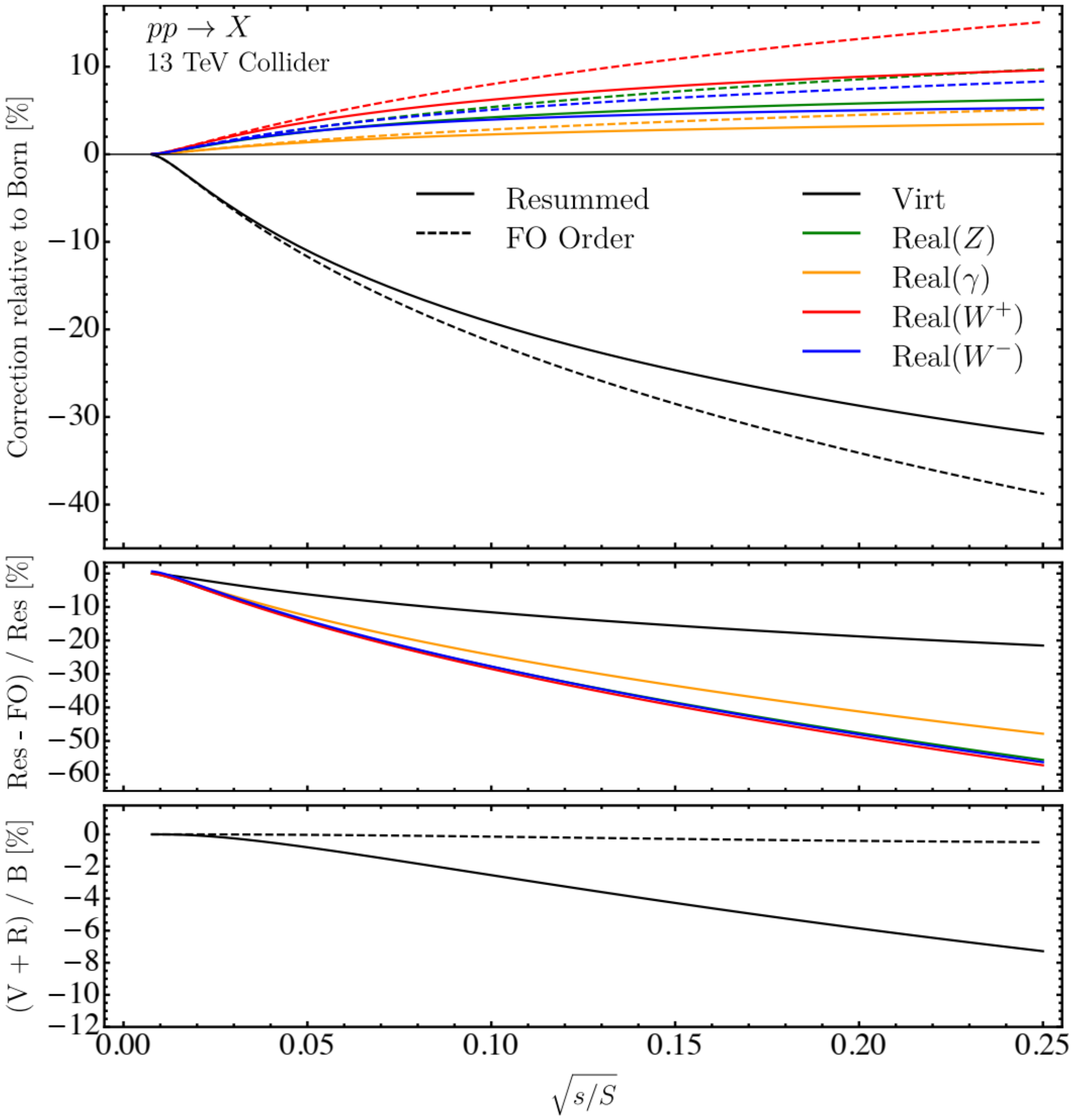
Significant corrections to many processes at high energies:

Exclusive di-jet: ~ 10-30%

Bell, Kuhn and Rittinger: 1004.4117

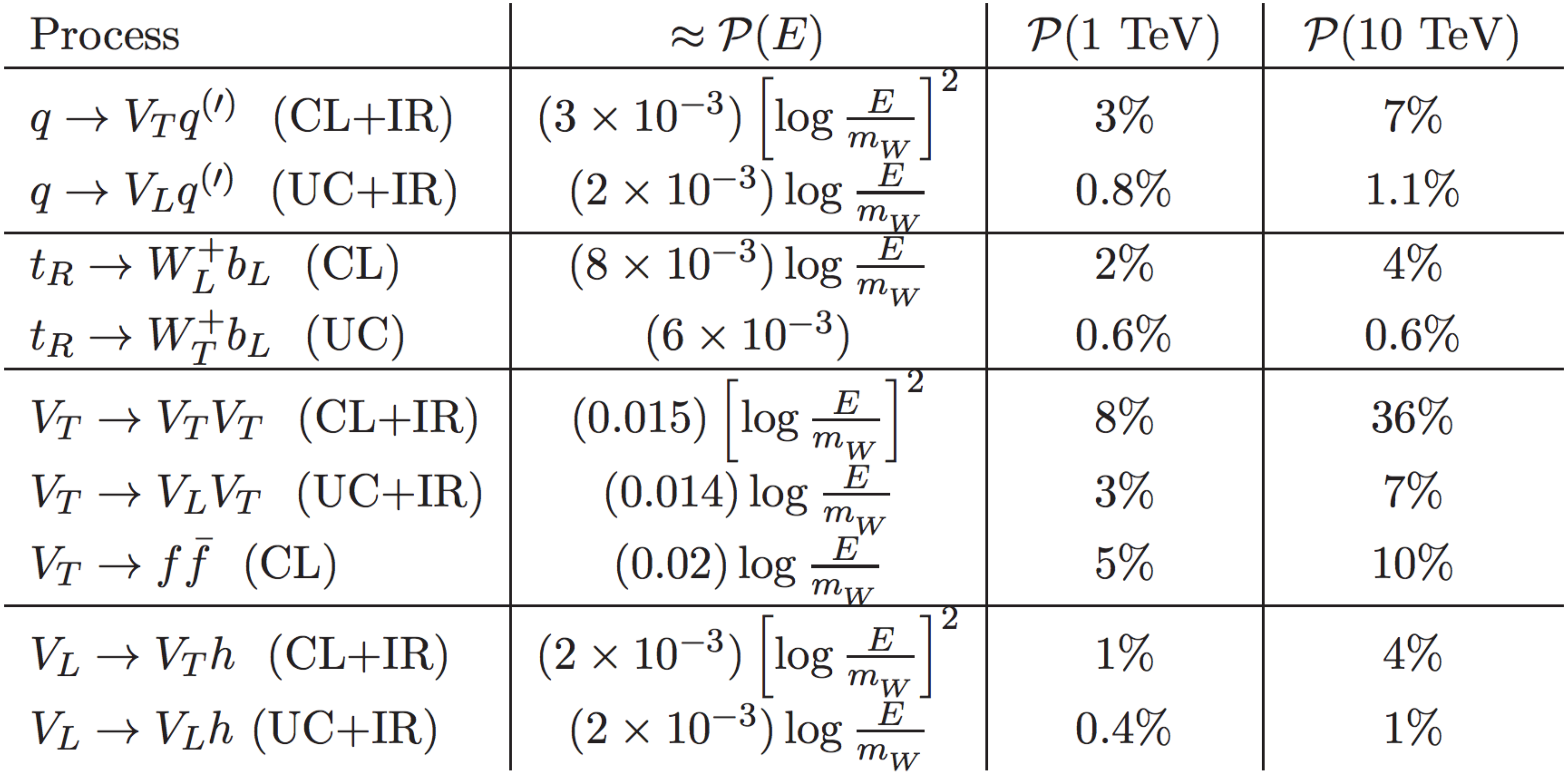
W/Z + jets: ~ 5-10%

Kuhn, Kulesza, Pozzorini, Schulze: 0703.283



Bauer, Ferland: 1601.07190

Importance of EW radiation



Process	$\approx \mathcal{P}(E)$	$\mathcal{P}(1\ \mathrm{TeV})$	$\mathcal{P}(10\ \mathrm{TeV})$
$q \to V_T q^{(\prime)}$ (CL+IR)	$(3 \times 10^{-3}) \left[\log \frac{E}{m_W}\right]^2$	3%	7%
$q \to V_L q^{(\prime)}$ (UC+IR)	$(2 \times 10^{-3}) \log \frac{E}{m_W}$	0.8%	1.1%
$t_R \to W_L^+ b_L$ (CL)	$(8 \times 10^{-3}) \log \frac{E}{m_W}$	2%	4%
$t_R \to W_T^+ b_L$ (UC)	(6×10^{-3})	0.6%	0.6%
$V_T \to V_T V_T$ (CL+IR)	$(0.015) \left[\log \frac{E}{m_W}\right]^2$	8%	36%
$V_T \to V_L V_T$ (UC+IR)	$(0.014) \log \frac{E}{m_W}$	3%	7%
$V_T \to f\bar{f}$ (CL)	$(0.02) \log \frac{E}{m_W}$	5%	10%
$V_L \to V_T h$ (CL+IR)	$(2 \times 10^{-3}) \left[\log \frac{E}{m_W}\right]^2$	1%	4%
$V_L \to V_L h$ (UC+IR)	$(2 \times 10^{-3}) \log \frac{E}{m_W}$	0.4%	1%

Chen, Han, Tweedie: 1611.00788

Complications for EW radiation

- CP violation $\longrightarrow$ forced to keep track of fermion helicities
- Mass effects of the gauge bosons show up

$$\Delta_i = 2p_i \cdot p_k + m_V^2$$

$$a_V^{\text{emit}} = \frac{2g_V^2}{s}(C_v - \lambda C_a)\left(\frac{(s-\Delta_a)(s-\Delta_b)}{\Delta_a \Delta_b} + (\Delta_a\Delta_b - m_V^2)\left(\frac{1}{\Delta_a^2} + \frac{1}{\Delta_b^2}\right)\right)$$

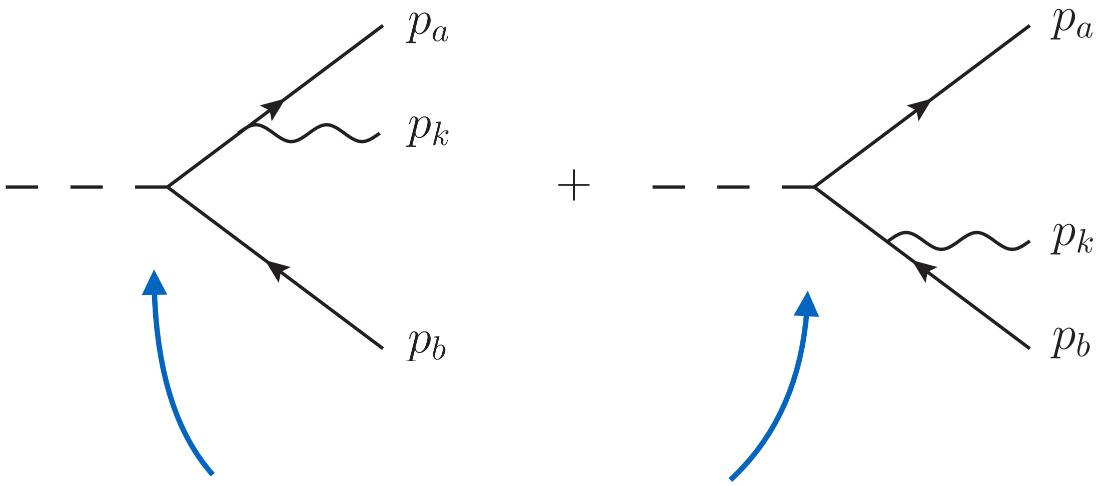
- Electroweak decays are a natural part of an EW parton shower

$$t \to Wb \quad Z \to f\bar{f} \quad W \to f\bar{f}'$$

- Massive fermions $\longrightarrow$ Helicity becomes handedness (not Lorentz invariant)
  $\longrightarrow$ Handedness can flip

- Physical differences between transverse and longitudinal gauge bosons
  $\longrightarrow$ Keep track of those as well

Amplitude level calculations



Vertex decides initial handedness configuration

$$V = \not{q}_1 u_{\rho_1}(p_A)\bar{v}_{\rho_2}(p_B)\not{q}_2$$

Polarization vectors

$$\epsilon_T^\mu = \frac{1}{\sqrt{2}m}\bar{u}_\pm(k_1)\gamma^\mu u_\pm(k_2)$$

$$\epsilon_L^\mu = \frac{1}{m}(k_1 - k_2)$$

Spinors

$$u_\lambda(p) = \frac{1}{\sqrt{2k_0 \cdot p}}(\not{p} + m)u_{-\lambda}(k_0)$$

$$v_\lambda(p) = \frac{1}{\sqrt{2k_0 \cdot p}}(\not{p} - m)u_{-\lambda}(k_0)$$

Write everything in terms of products of spinors
$\rightarrow$ Easily calculable

Future: More than two fermions → Reduction of computation times

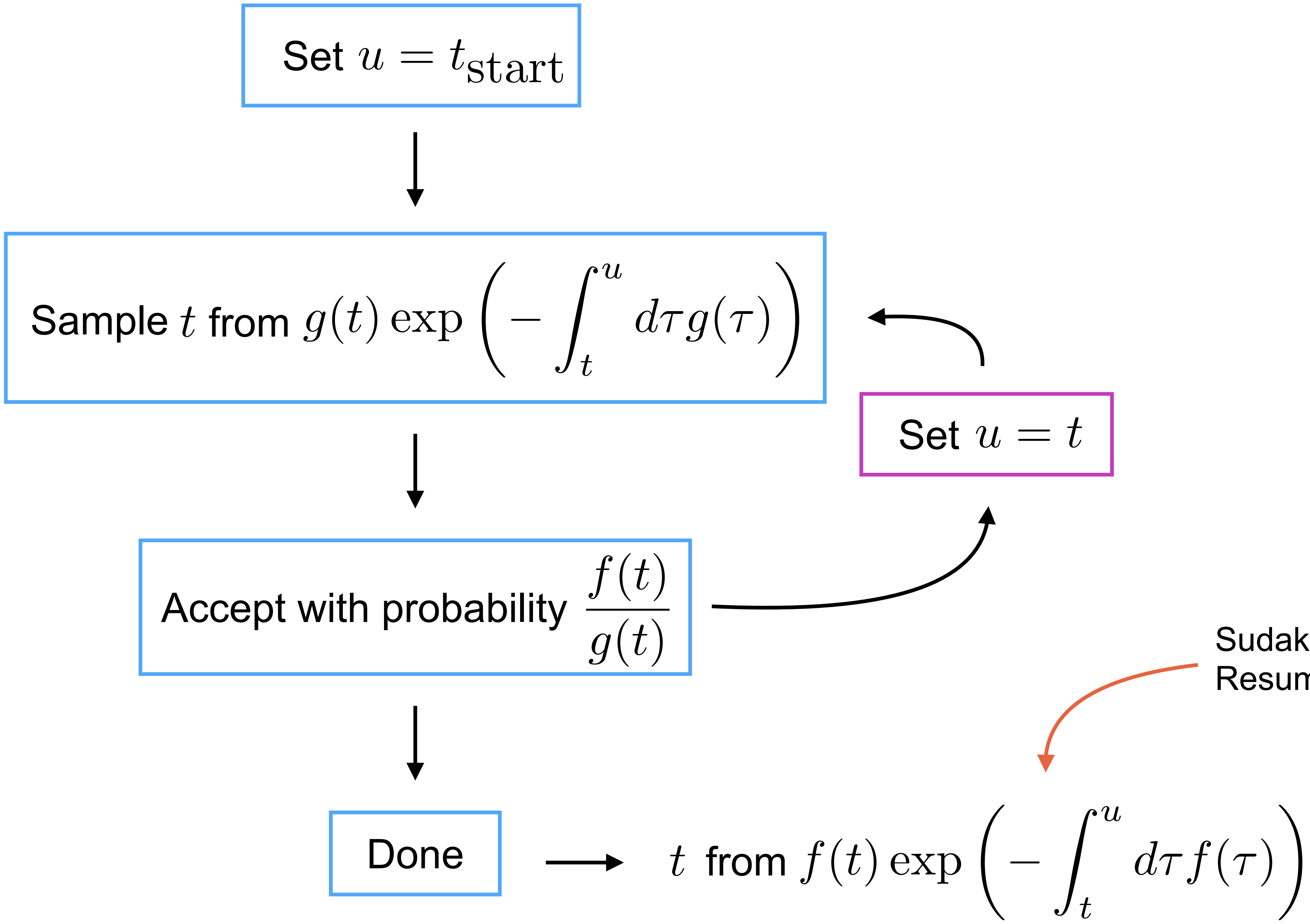
Conclusion & Outlook

Photon emission
- Resums soft and collinear logarithms
- Fully coherent

Photon splitting
- Resums collinear logarithms
- Corrects for on-shell photon effects

Electroweak radiation
- Complications due to mass and helicities
- Naturally incorporates electroweak decays
- Amplitude level calculations

Sudakov Veto Algorithm



Set $u = t_{\text{start}}$

Sample t from $g(t)\exp\left(-\int_t^u d\tau\, g(\tau)\right)$

Accept with probability $\frac{f(t)}{g(t)}$

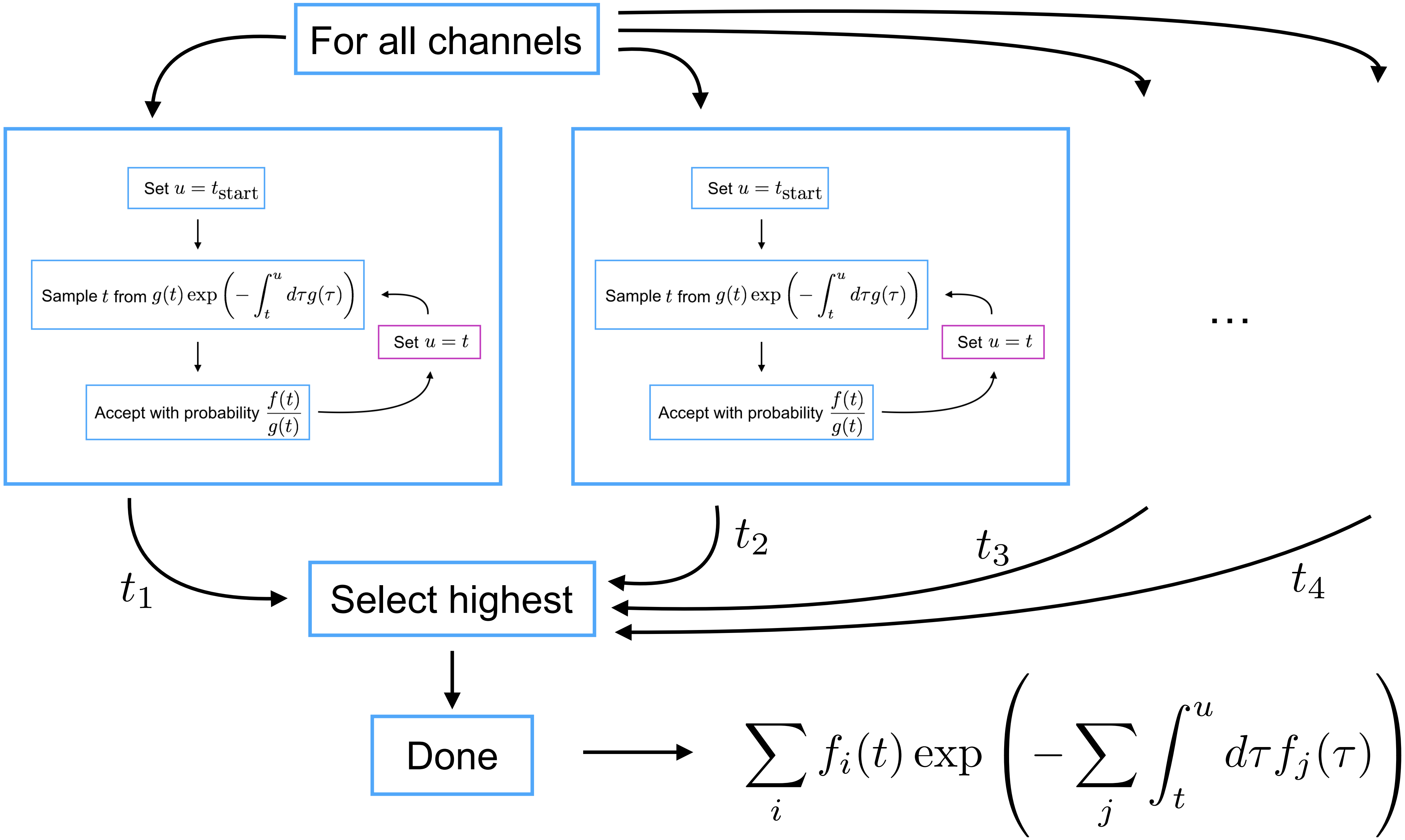
Set $u = t$

Done $\longrightarrow$ t from $f(t)\exp\left(-\int_t^u d\tau\, f(\tau)\right)$

Sudakov form factor
Resums logarithm

Sudakov Veto Algorithm - Competition 1

Multiple channels $g_i(t) > f_i(t)$



For all channels

Set $u = t_{\text{start}}$

Sample t from $g(t) \exp\left(-\int_t^u d\tau g(\tau)\right)$

Set $u = t$

Accept with probability $\frac{f(t)}{g(t)}$

Set $u = t_{\text{start}}$

Sample t from $g(t) \exp\left(-\int_t^u d\tau g(\tau)\right)$

Set $u = t$

Accept with probability $\frac{f(t)}{g(t)}$

...

t_1

t_2

t_3

t_4

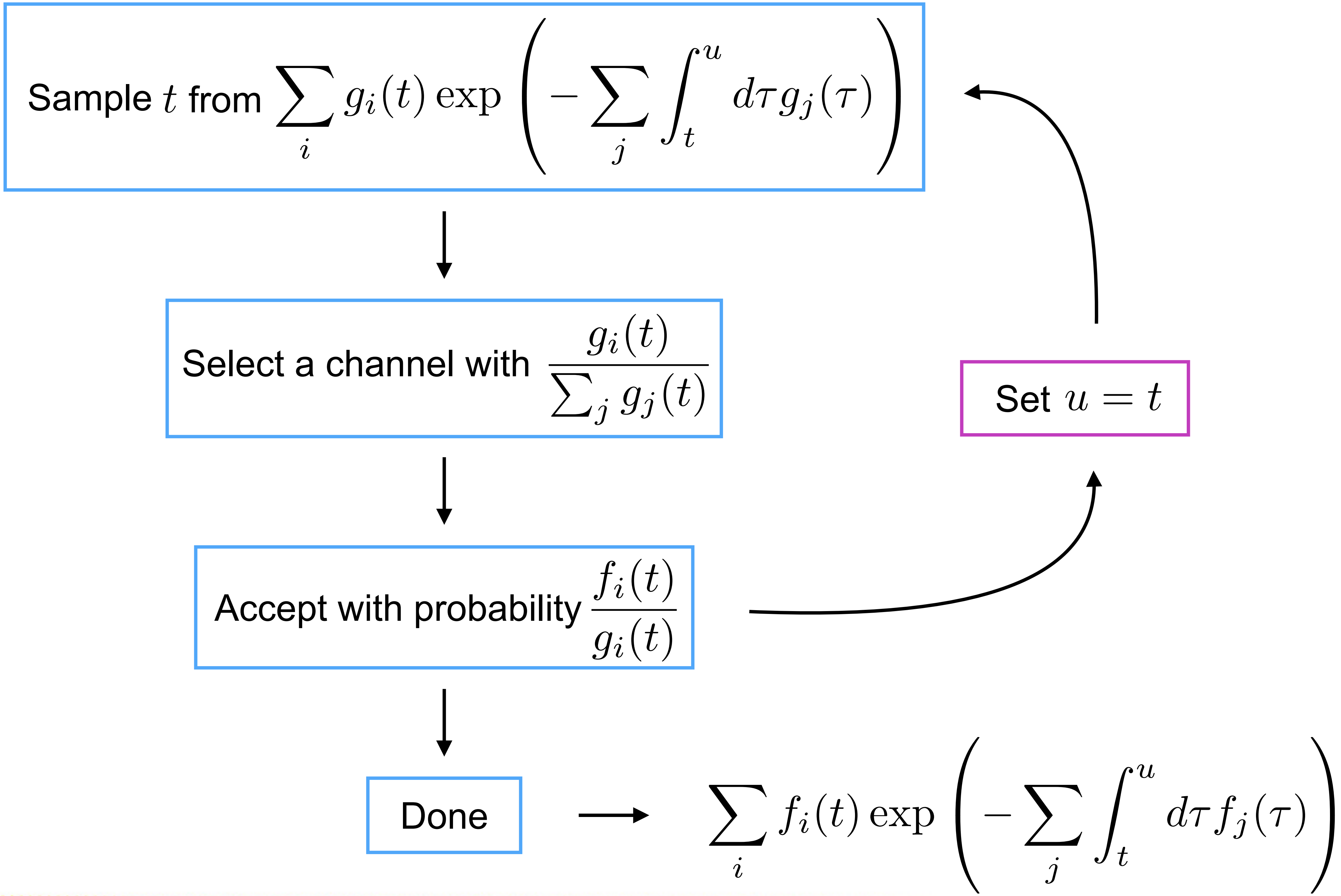
Select highest

Done

$$\longrightarrow \sum_i f_i(t) \exp\left(-\sum_j \int_t^u d\tau f_j(\tau)\right)$$

Sudakov Veto Algorithm - Competition 2

Kleiss, Verheyen: 1605.09246



Sample t from $\sum_i g_i(t) \exp\left(-\sum_j \int_t^u d\tau\, g_j(\tau)\right)$

Select a channel with $\dfrac{g_i(t)}{\sum_j g_j(t)}$

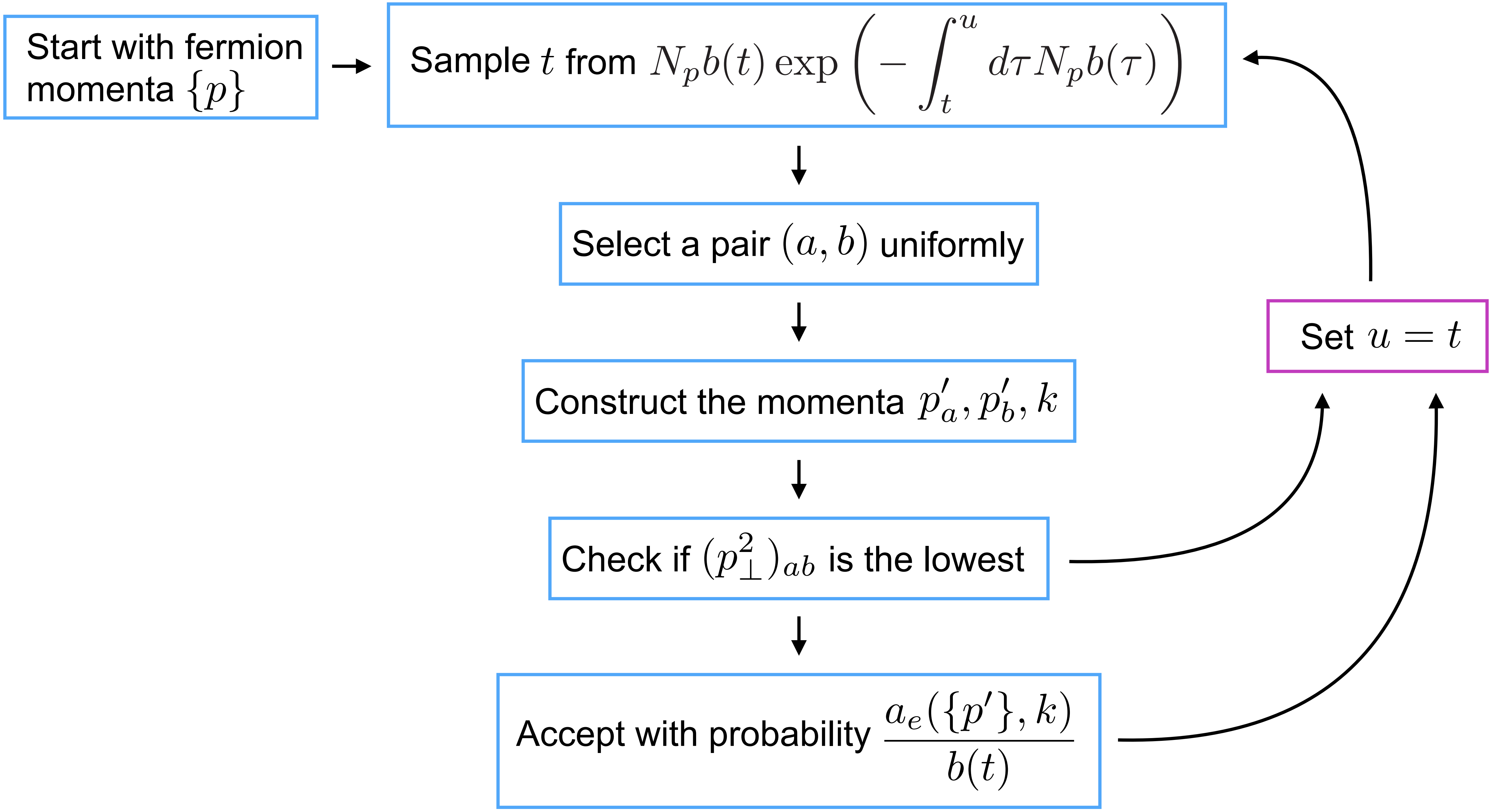
Accept with probability $\dfrac{f_i(t)}{g_i(t)}$

Set $u = t$

Done $\longrightarrow$ $\sum_i f_i(t) \exp\left(-\sum_j \int_t^u d\tau\, f_j(\tau)\right)$

Sudakov Veto Algorithm - Photon Emission

Find an overestimate $b(t)$ of $a_e^{QED}(\{p\}, k)$ **(simplified)**



Start with fermion momenta $\{p\}$

Sample t from $N_p b(t) \exp\left(-\int_t^u d\tau N_p b(\tau)\right)$

Select a pair (a, b) uniformly

Construct the momenta p'_a, p'_b, k

Check if $(p_\perp^2)_{ab}$ is the lowest

Accept with probability $\dfrac{a_e(\{p'\}, k)}{b(t)}$

Set $u = t$

Introduction

Two approaches to QED radiation in parton showers

DGLAP

- Resums collinear photon logarithms
- Interleaving with QCD shower
- Also applicable in antenna/dipole showers

YFS

- Resums soft photon logarithms
- Collinear logarithms can be included, but not resummed
- Afterburner to add soft photons

Can we resum both the soft and collinear logarithms?

Follow QCD antenna shower Vincia

Giele, Kosower, Skands:1102.2126

Gehrmann, Ritzmann, Skands:1108.6172