Global Recoil in Initial-Final Antennae

Rob Verheyen

With Peter Skands & Ronald Kleiss

IMAPP
Nikhef
Radboud University

Introduction

Parton showers simulate initial state radiation (ISR) with backward evolution

Two types of showers:

- DGLAP-style with $1 \to 2$ branchings
  - Pythia, Herwig(angular)
  - Recoil from ISR shared globally
- Dipole/antenna-style with $2 \to 3$ branchings
  - Sherpa, Herwig(dipole), Dire, Vincia
  - Recoil from ISR shared:
    1. Globally for initial-initial connections
    2. Locally for initial-final connections

Goal: Find a way for initial-final connections to share recoil globally

Context: Vincia, a shower plugin for Pythia based on antenna factorization

Giele, Kosower, Skands:1102.2126

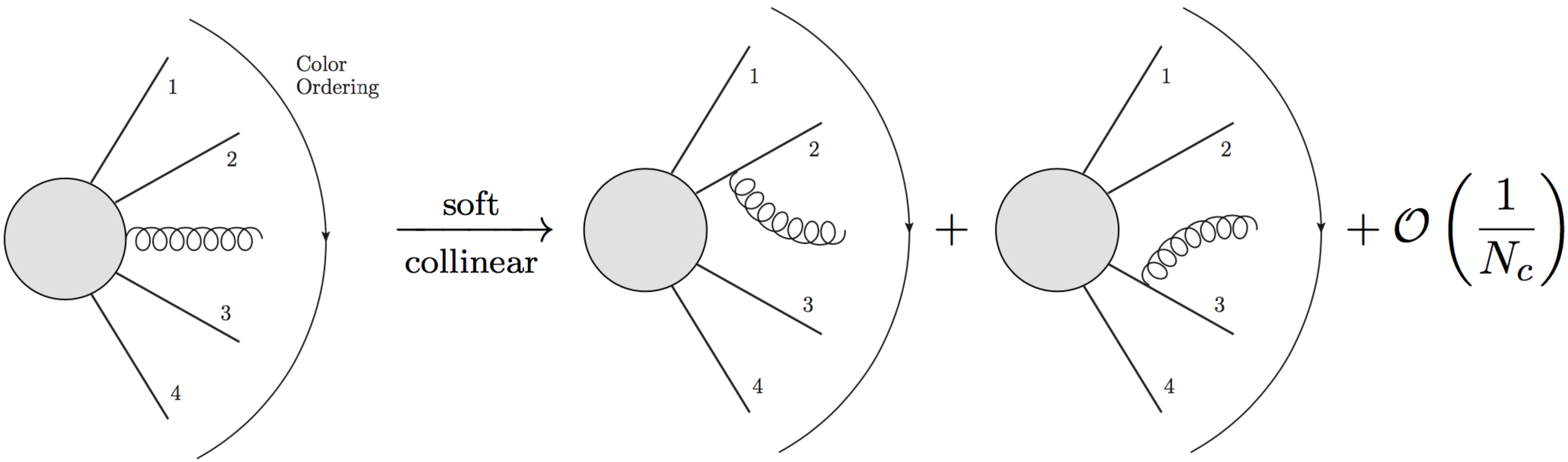
Gehrmann, Ritzmann, Skands:1108.6172

Antenna Factorization

Antenna Factorization

$$|M(.., p_a, k, ..)|^2 \xrightarrow{p_a \| k} g^2 C \frac{P(z)}{p_a \cdot k} |M(.., p_a + k, ..)|^2$$

$$|M(.., p_a, k, p_b, ..)|^2 \xrightarrow{k \to 0} g^2 C \left[\frac{2 p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} - \frac{m_a^2}{(p_a \cdot k)^2} - \frac{m_b^2}{(p_b \cdot k)^2} \right] |M(.., p_a, p_b, ..)|^2$$



Antenna Factorization

Antenna Factorization

$$|M(..,p_a,k,p_b,..)|^2 \approx g^2 C\, a_e^{QCD}(p_a,k,p_b)|M(..,p'_a,p'_b,..)|^2$$

Antenna functions are similar to Catani-Seymour dipoles

$$a_e^{QCD} \approx D_{ak,b}^{CS} + D_{bk,a}^{CS}$$

Collinear behaviour is split between dipoles
Add up to correct soft behaviour

Phase Space Factorization

Major advantage of the 2 $\rightarrow$ 3 scheme:

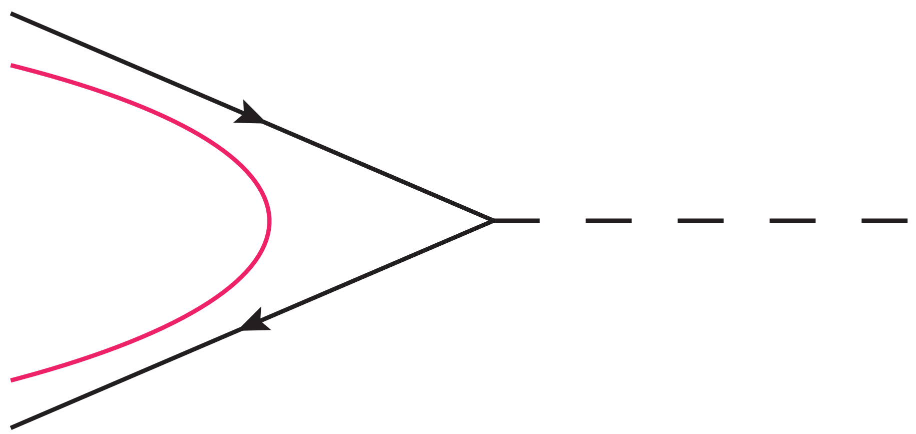
Exact factorization of phase space

$$\frac{dx_a}{x_a}\frac{dx_b}{x_b}d\Phi_n = \frac{dx_A}{x_A}\frac{dx_B}{x_B}d\Phi_{n-1}d\Phi_{\mathrm{ant}}$$

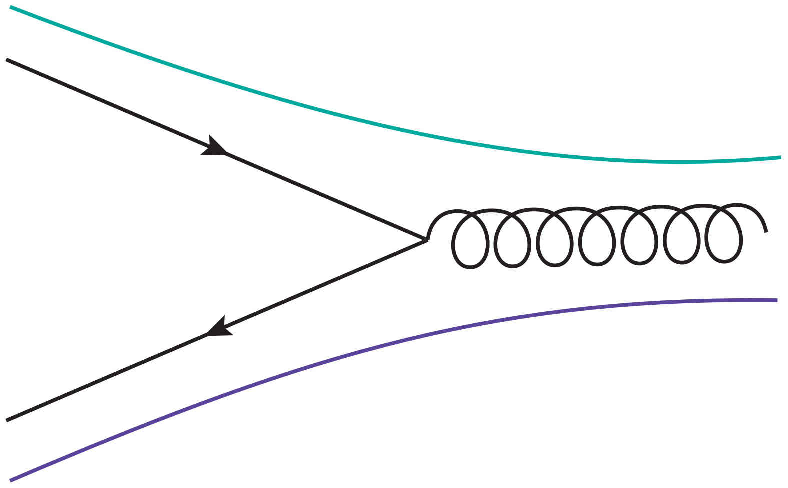
On-shell momenta with exact momentum conservation

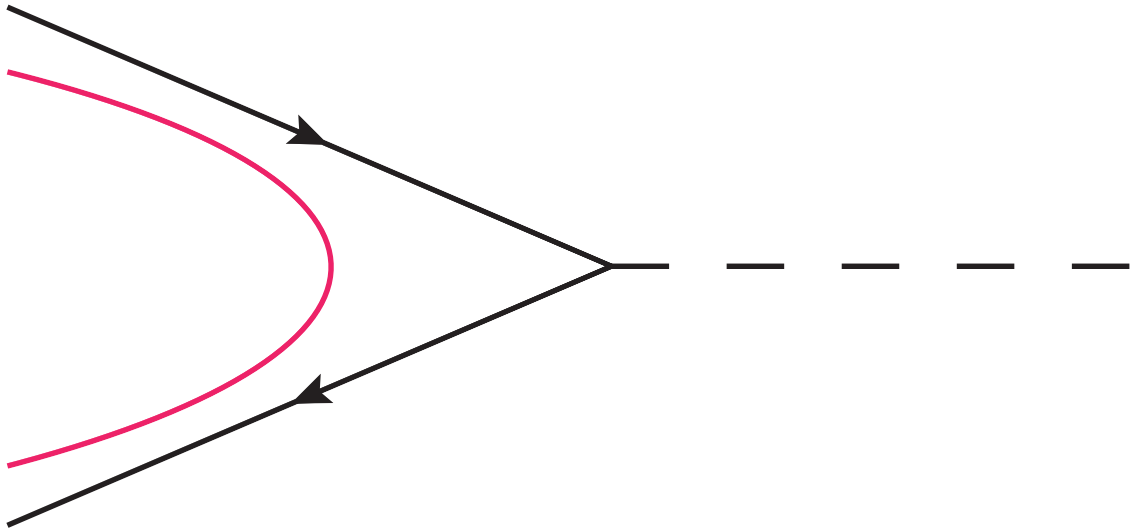
Two scenarios:

Initial-initial



Initial-final





Initial-Initial

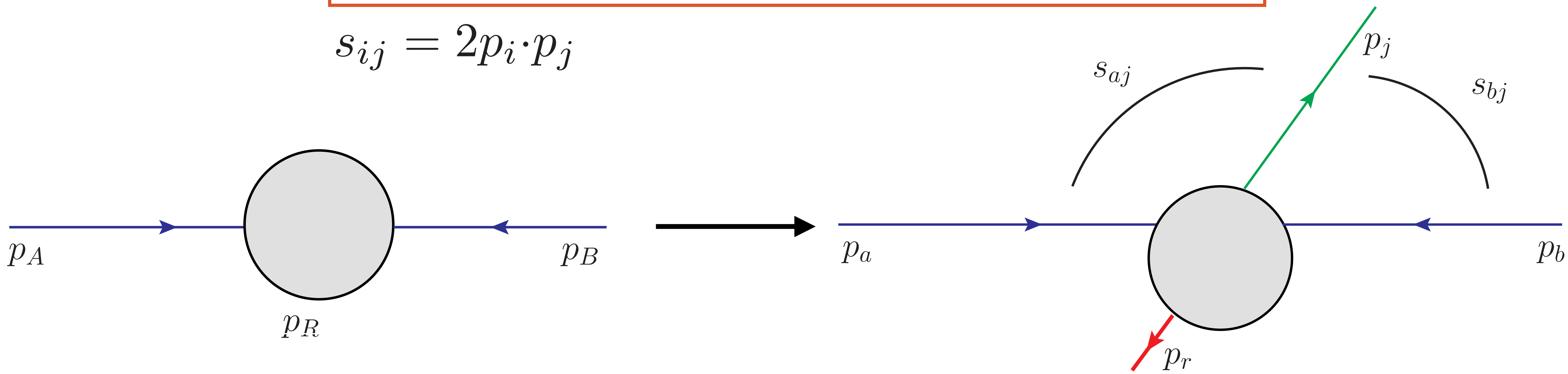
Phase Space Factorization - II

$$\frac{dx_a}{x_a}\frac{dx_b}{x_b}d\Phi_2(p_a+p_b\to p_j+p_r)$$
$$=\frac{dx_A}{x_A}\frac{dx_B}{x_B}d\Phi_1(p_A+p_B\to p_R)\,d\Phi_{\text{ant}}$$

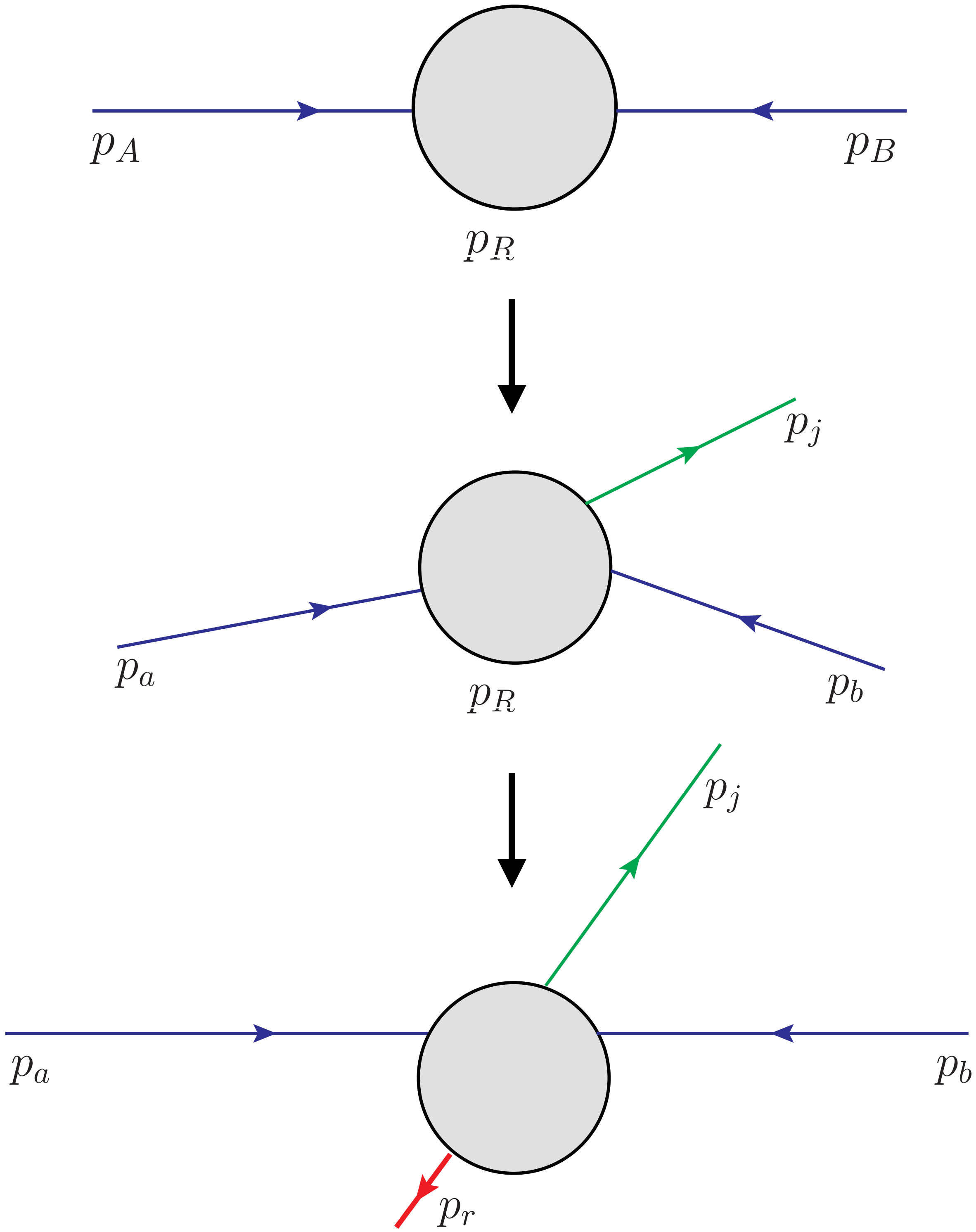
$$x_I < x_i < 1$$

$$d\Phi_{\text{ant}} = \frac{1}{16\pi^2}\frac{1}{s_{AB}}\left(\frac{x_A}{x_a}\frac{x_B}{x_b}\right)^2 ds_{aj}ds_{bj}\frac{d\varphi}{2\pi}$$

$$s_{ij} = 2p_i{\cdot}p_j$$



Phase Space Factorization - II



Shortcut to avoid doing Lorentz transform

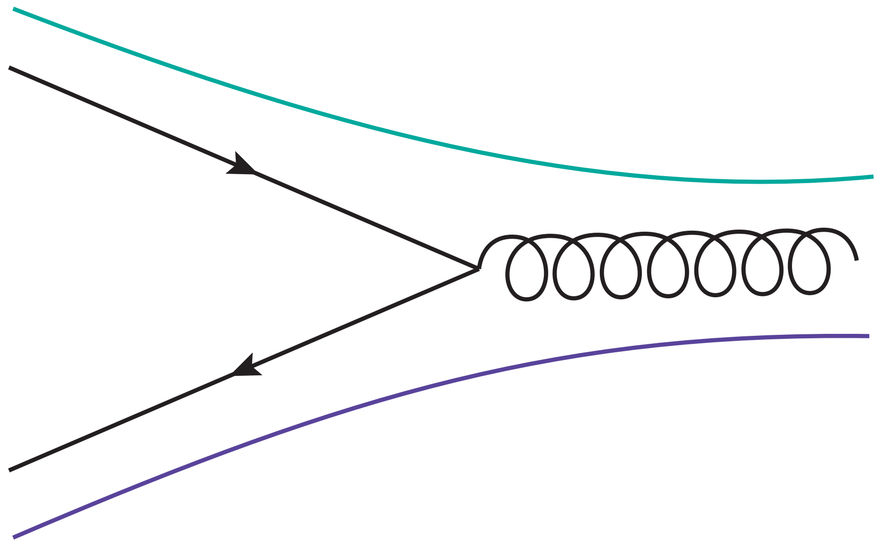
$$p_a = c_1 p_A$$

$$p_b = c_2 p_B$$

$$p_j = c_3 p_A + c_4 p_B + c_5 p_\perp(\varphi)$$

$$p_r = p_a + p_b - p_j$$

- Recoil on rest of the system is required
- Single free parameter



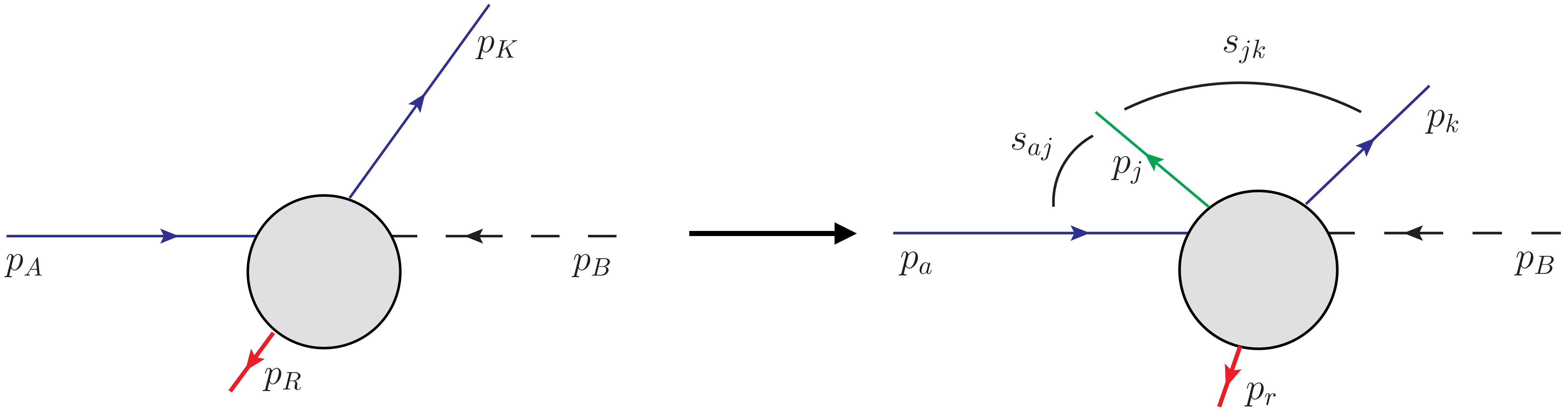
Initial-Final

Radboud University

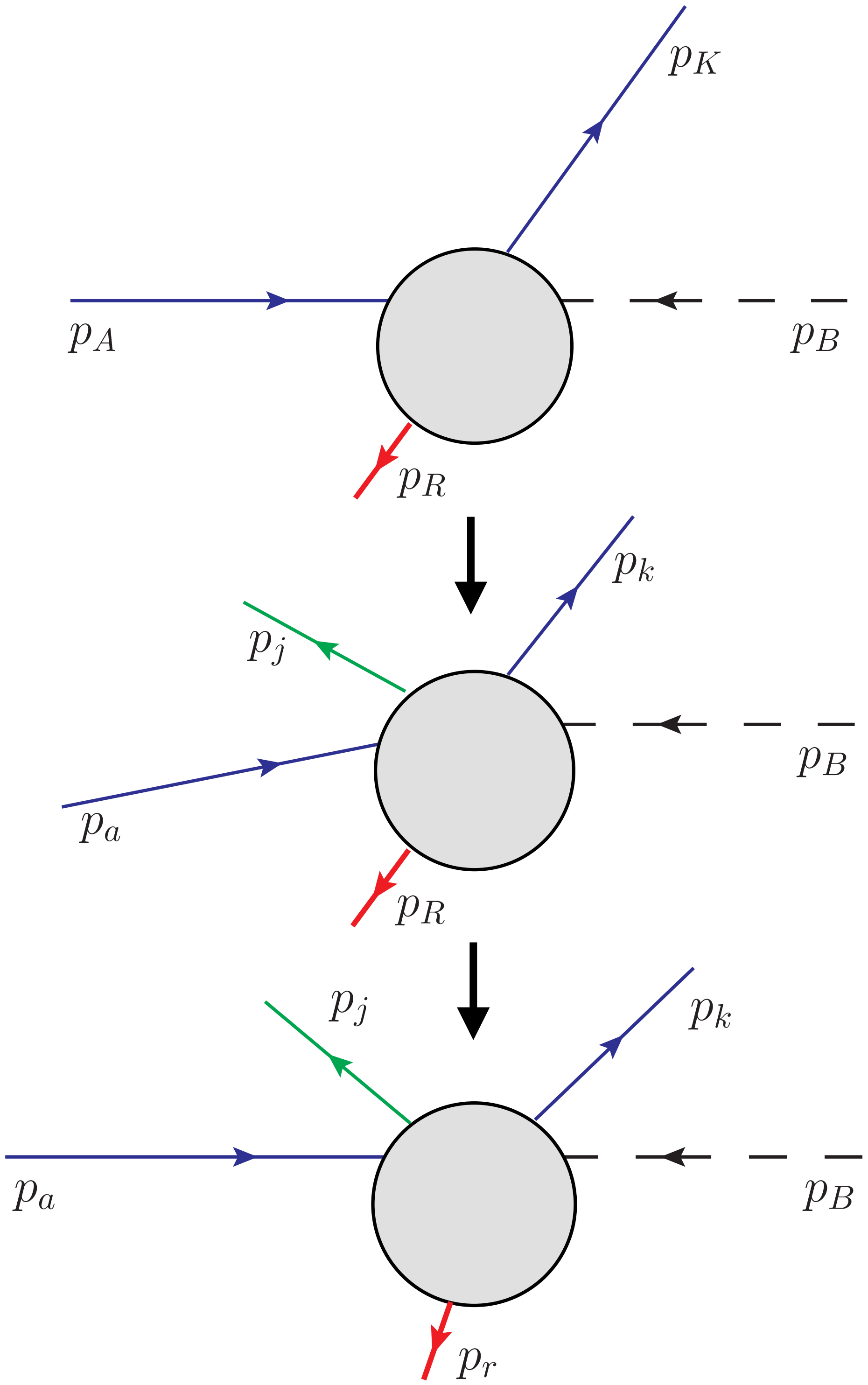
Phase Space Factorization - IF

$$\frac{dx_a}{x_a}\frac{dx_B}{x_B}d\Phi_3(p_a + p_B \to p_j + p_k + p_r)$$
$$= \frac{dx_A}{x_A}\frac{dx_B}{x_B}d\Phi_1(p_A + p_B \to p_K + p_R)\, d\Phi_{\text{ant}}$$

$$d\Phi_{\text{ant}} = \frac{1}{16\pi^2}\frac{1}{s_{AK}}\frac{x_A^2}{x_a^2}ds_{aj}ds_{jk}\frac{d\varphi}{2\pi}$$



Phase Space Factorization - IF



Try the same shortcut

Kinematics

$$p_a = c_1 p_A$$

$$p_j = c_2 p_A + c_3 p_K + c_4 p_R + c_5 p_\perp(\varphi)$$

$$p_k = c_6 p_A + c_7 p_K + c_8 p_R + c_9 p_\perp(\varphi)$$

$$p_r = p_a - p_A + p_K + p_R - p_j - p_k$$

- Too many free parameters
- Makes little sense physically

Initial - Final Mapping

Map 1: p_A retains its direction

$$p_a = \frac{s_{AK} + s_{jk}}{s_{jk}} p_A$$

$$p_j = \frac{s_{jk} s_{ak}}{s_{AK}(s_{AK} + s_{jk})} p_A + \frac{s_{aj}}{s_{AK} + s_{jk}} p_K + \frac{\sqrt{s_{jk} s_{aj} s_{ak}}}{s_{AK} + s_{jk}} p_\perp(\varphi)$$

$$p_k = p_a - p_A + p_K - p_j$$

p_k emits p_j with p_a spectating

- No Lorentz boost required $\rightarrow$ p_R does not change
- Automatically $x_a > x_A$
- Correct collinear and soft behaviour
- Default map for dipole/antenna showers

Arbitrary difference between
- Initial-initial: Global recoil
- Initial-final: No global recoil

Initial - Final Mapping

Map 2: p_K retains its direction

$$p_a = \frac{s_{ak}}{s_{AK} - s_{aj}} p_A + \frac{s_{aj} s_{sjk}}{s_{AK}(s_{AK} - s_{aj})} p_K + \frac{\sqrt{s_{jk} s_{aj} s_{ak}}}{s_{AK} - s_{aj}} p_\perp(\varphi)$$

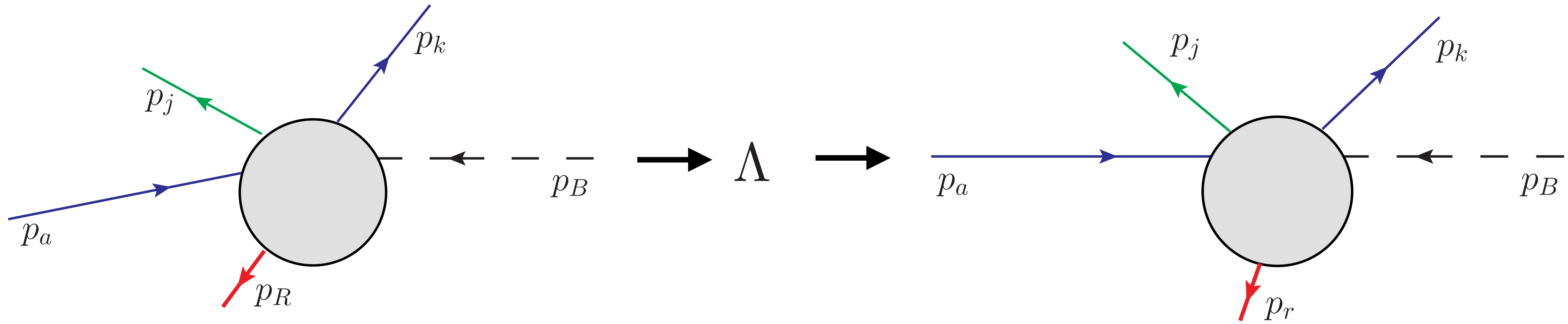
$$p_j = p_a - p_A + p_K - p_k$$

$$p_k = \frac{s_{AK} - s_{aj}}{s_{AK}} p_K$$

p_a emits p_j with p_k spectating

- Lorentz boost required to reallign $p_a \to p_R$ changes
- Not necessarily $x_a > x_A$, implemented with a veto
- Correct collinear and soft behaviour

Lorentz boost



1. Boost to rest frame of p_a and p_B
2. Rotate to beam axis
3. Boost to lab frame

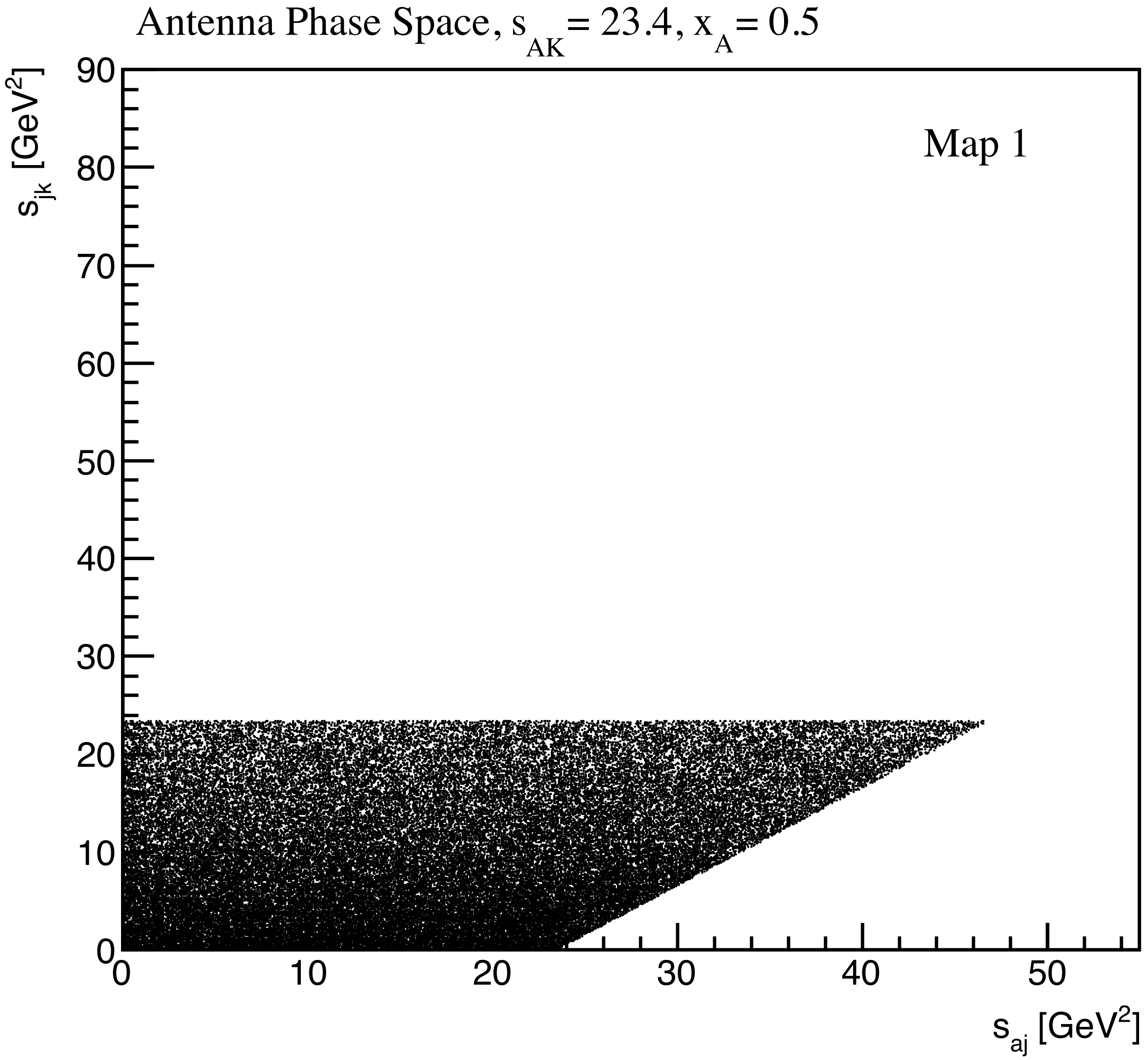
$$\Lambda^{\mu\nu} = g^{\mu\nu} + \frac{p_B^\mu p_a^\nu - p_a^\mu p_B^\nu}{p_a \cdot p_B} + \frac{p_A^\mu p_B^\nu - p_B^\mu p_A^\nu}{p_A \cdot p_B} + \frac{p_A \cdot p_a}{(p_A \cdot p_B)(p_a \cdot p_B)} p_B^\mu p_B^\mu$$

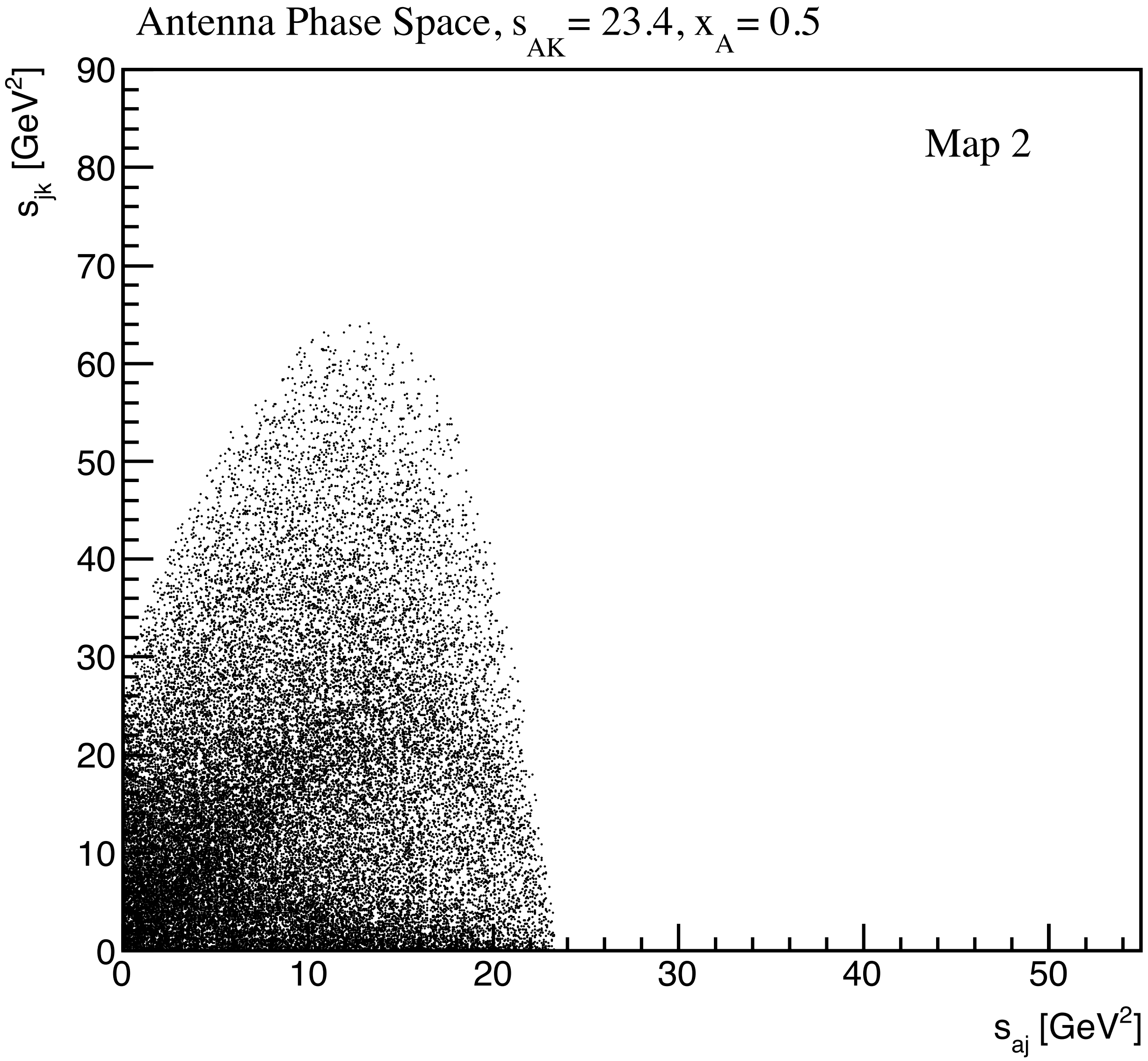
Properties: $(\Lambda p_B) = p_B$

$$(\Lambda p_a) = \frac{x_a}{x_A} p_A$$

Phase Space

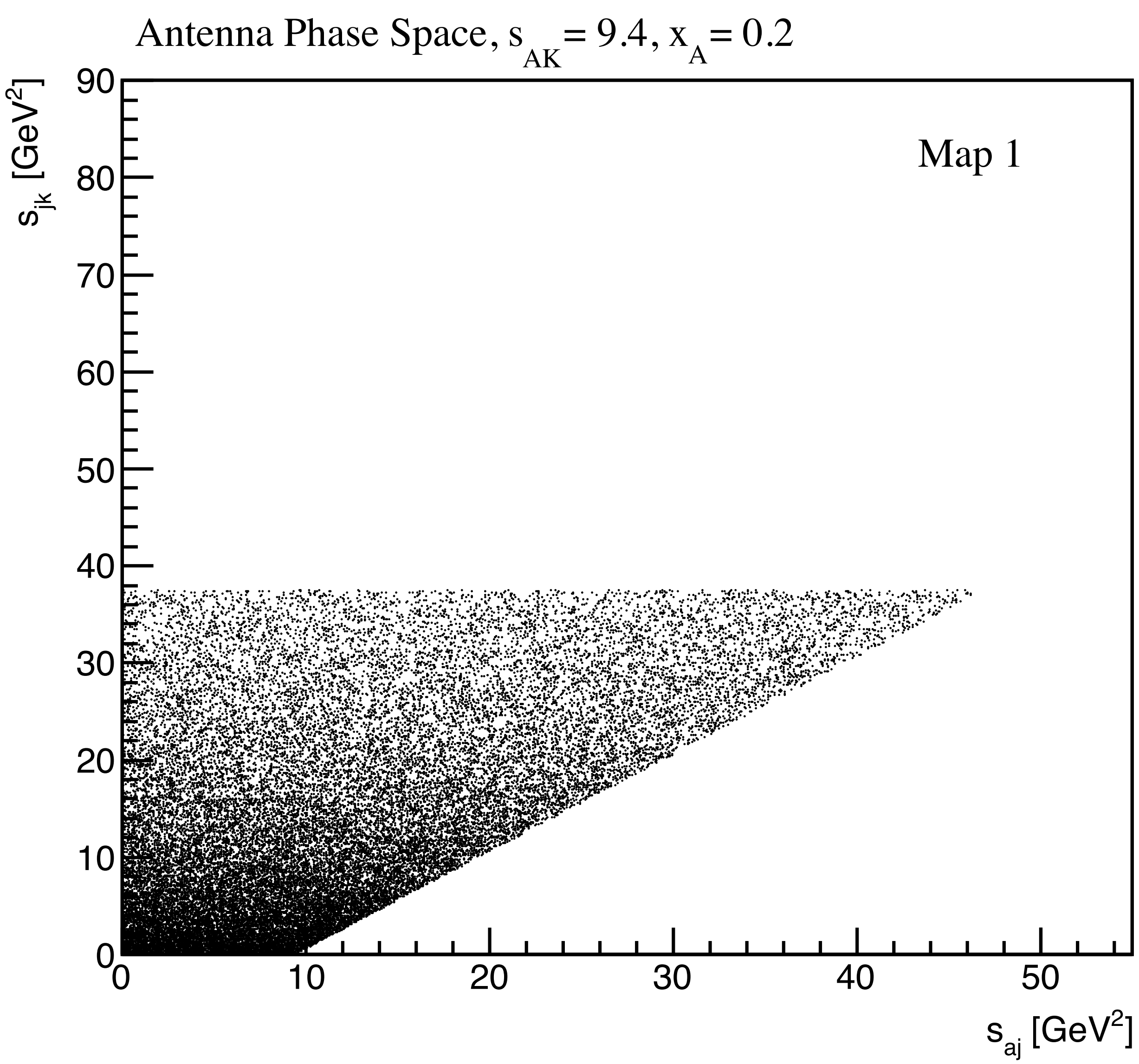
$$d\Phi_{\rm ant} = \frac{1}{16\pi^2}\frac{1}{s_{AK}}\frac{x_A^2}{x_a^2}ds_{aj}ds_{jk}\frac{d\varphi}{2\pi}$$

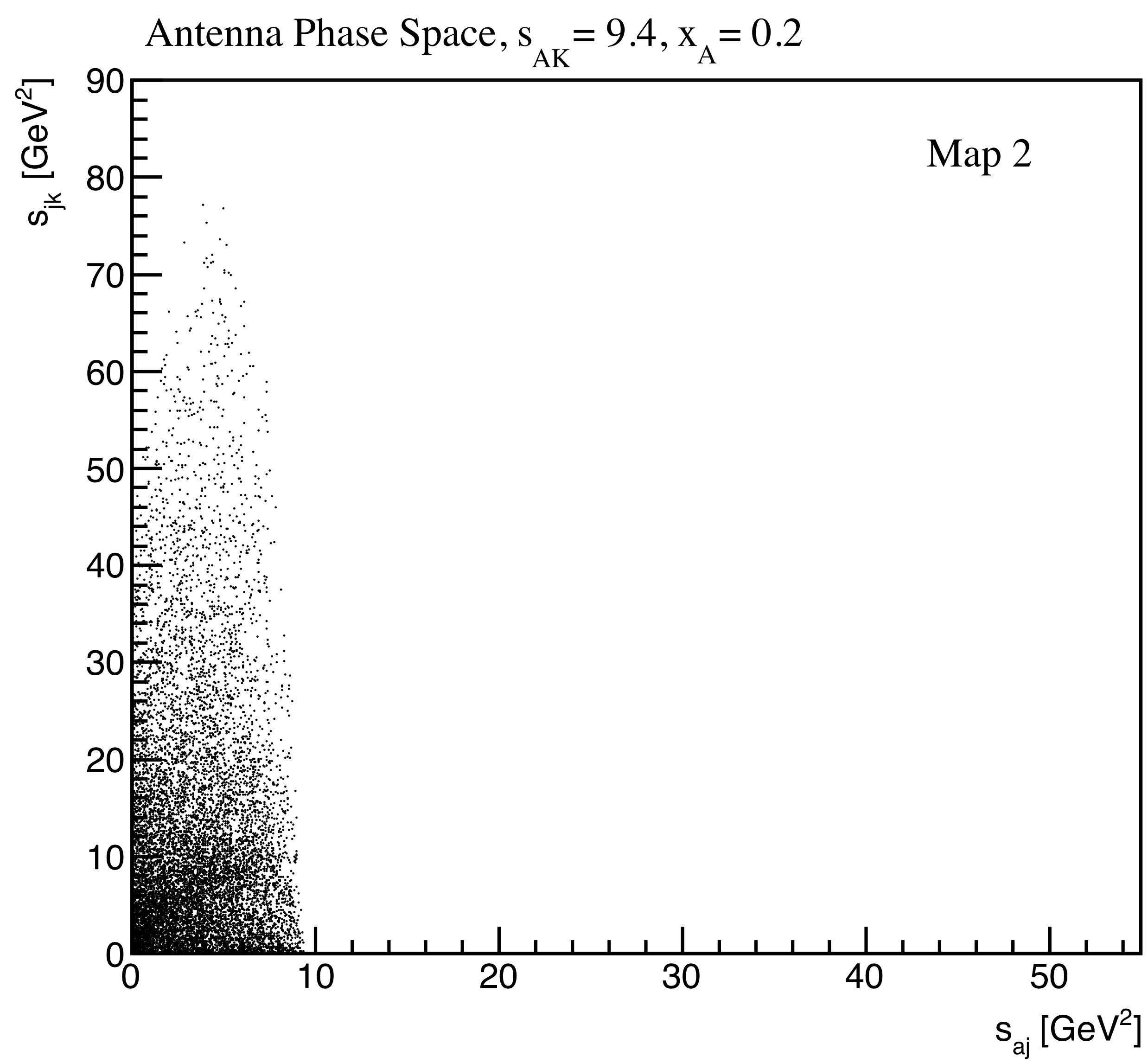




Phase Space

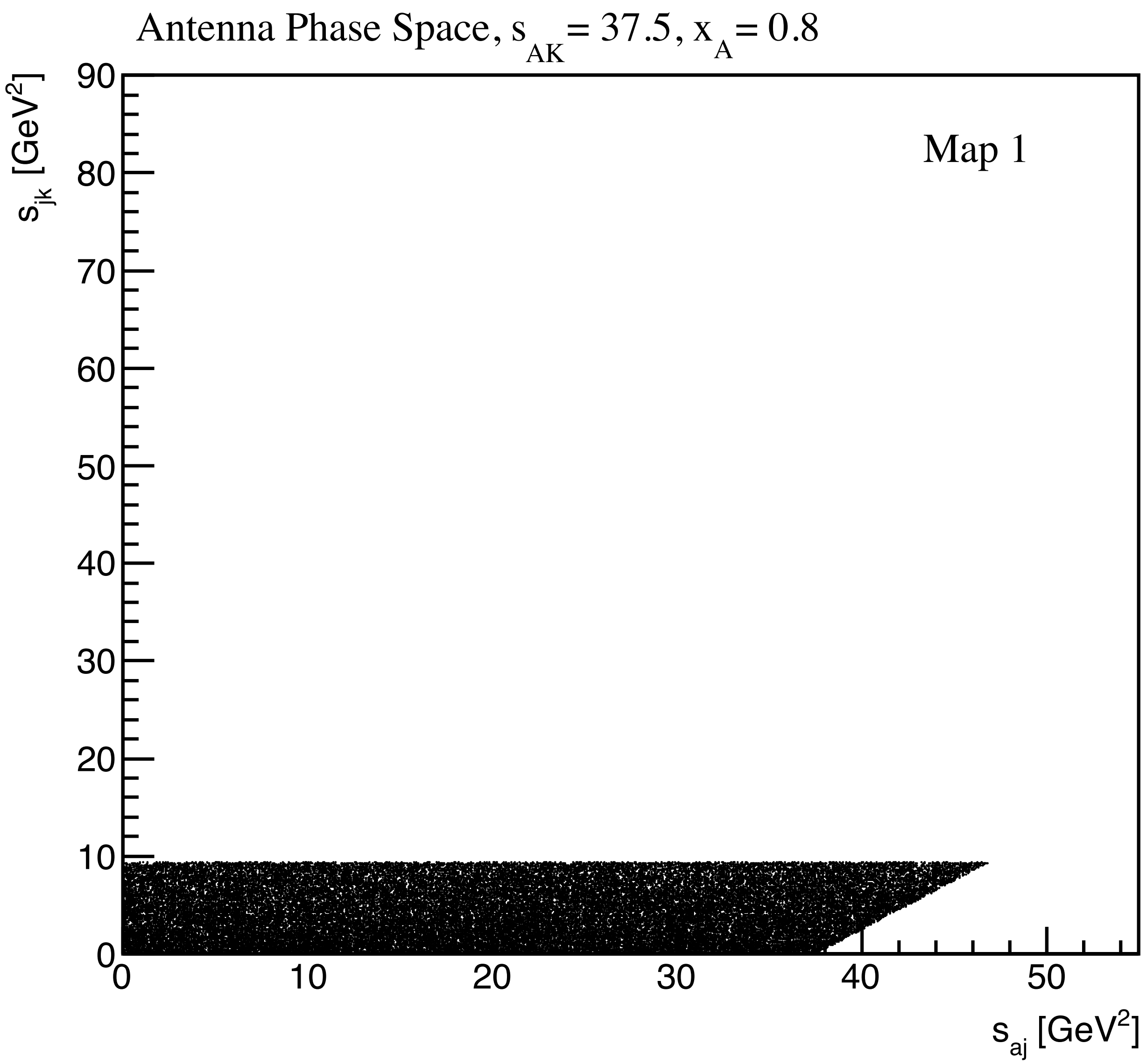
$$d\Phi_{\text{ant}} = \frac{1}{16\pi^2}\frac{1}{s_{AK}}\frac{x_A^2}{x_a^2}ds_{aj}ds_{jk}\frac{d\varphi}{2\pi}$$

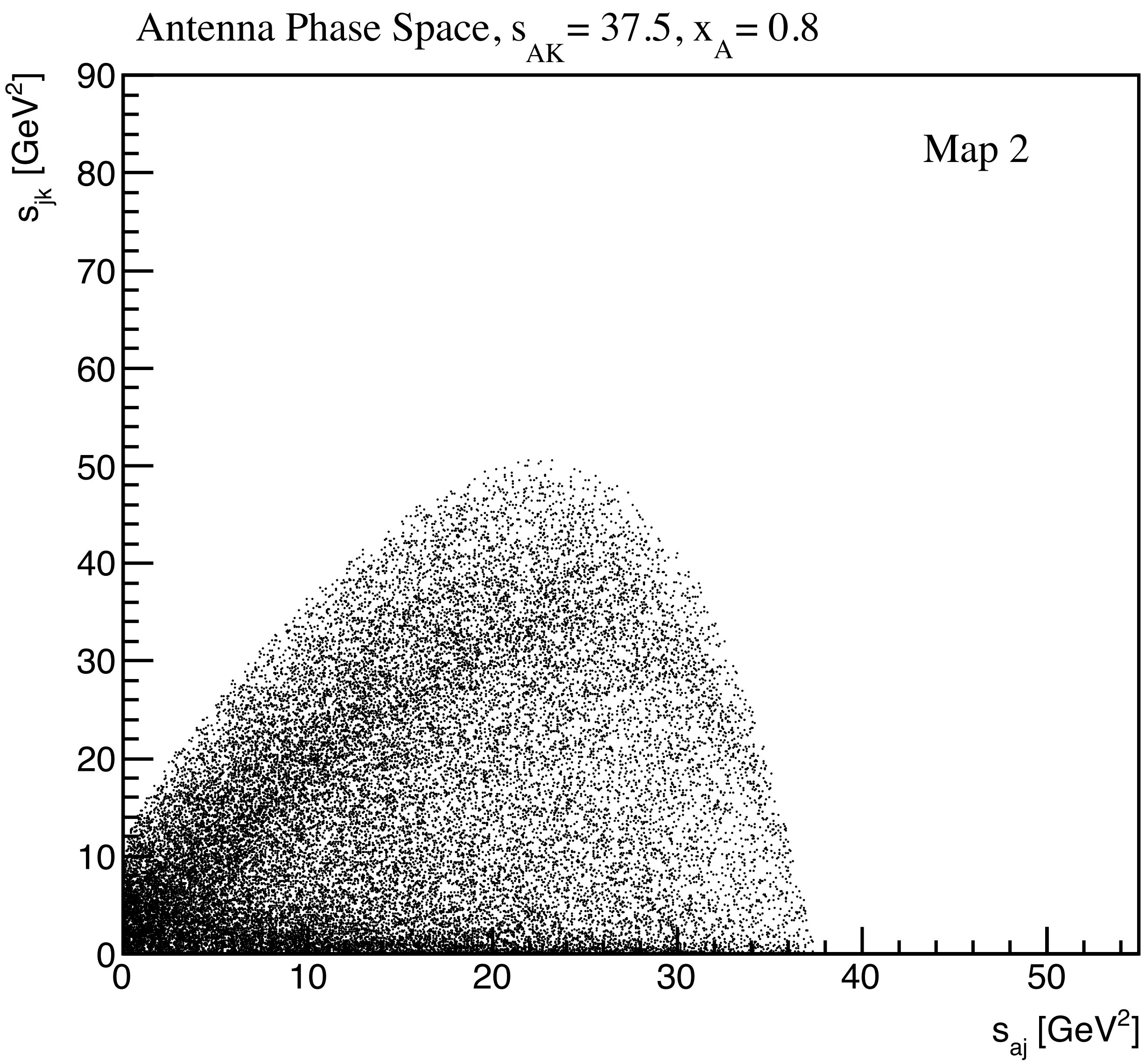




Phase Space

$$d\Phi_{\rm ant} = \frac{1}{16\pi^2}\frac{1}{s_{AK}}\frac{x_A^2}{x_a^2}ds_{aj}ds_{jk}\frac{d\varphi}{2\pi}$$





Implementation

Combine maps probabilistically

$$P_1 = \frac{s_{aj}}{s_{aj} + s_{jk}}$$

- Physically sensible choice
- Very similar to dipole approach

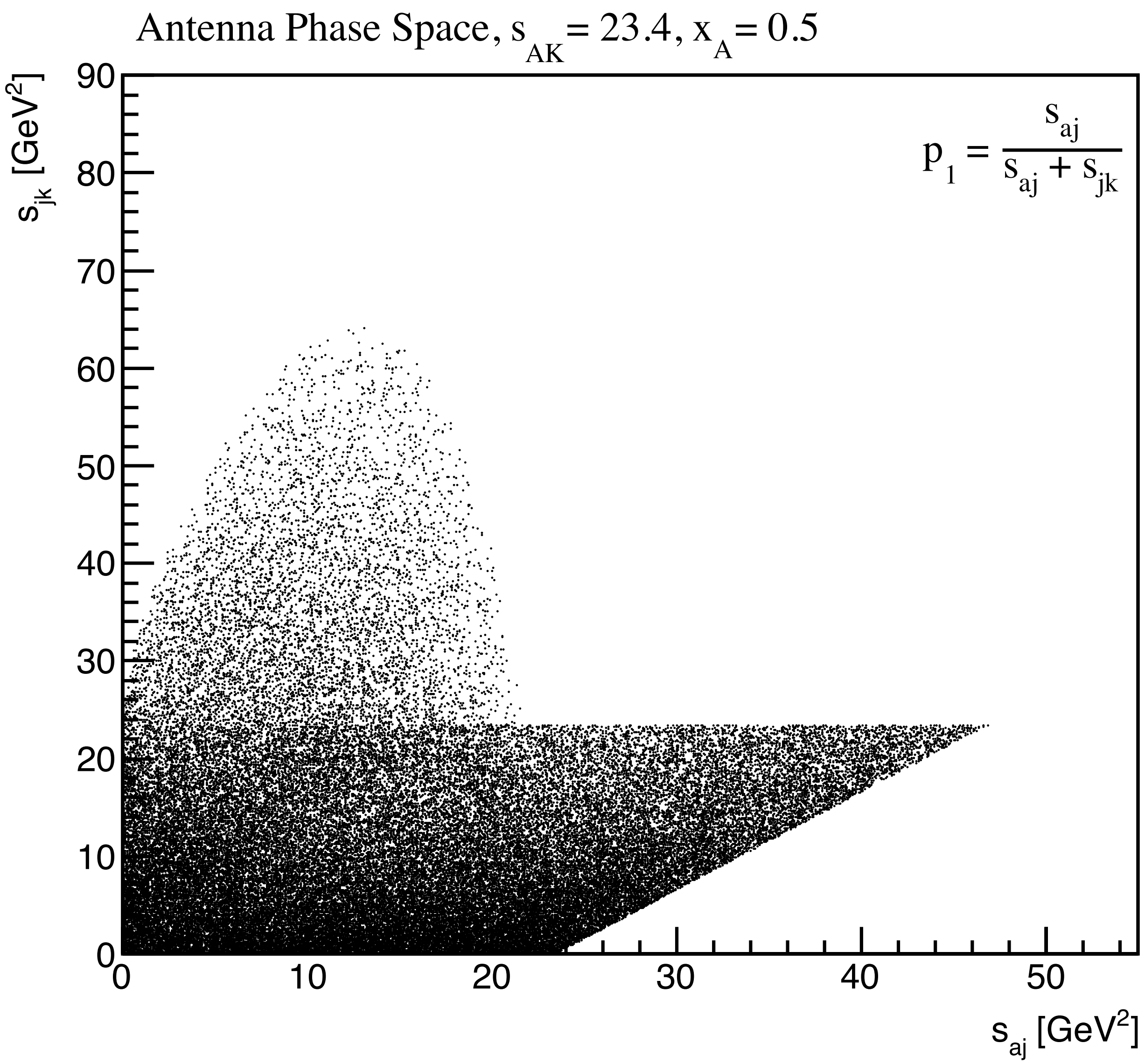
$$P_1 = \frac{s_{jk}}{s_{aj} + s_{jk}}$$

- Smaller coverage of phase space
- Still correct in IR limits

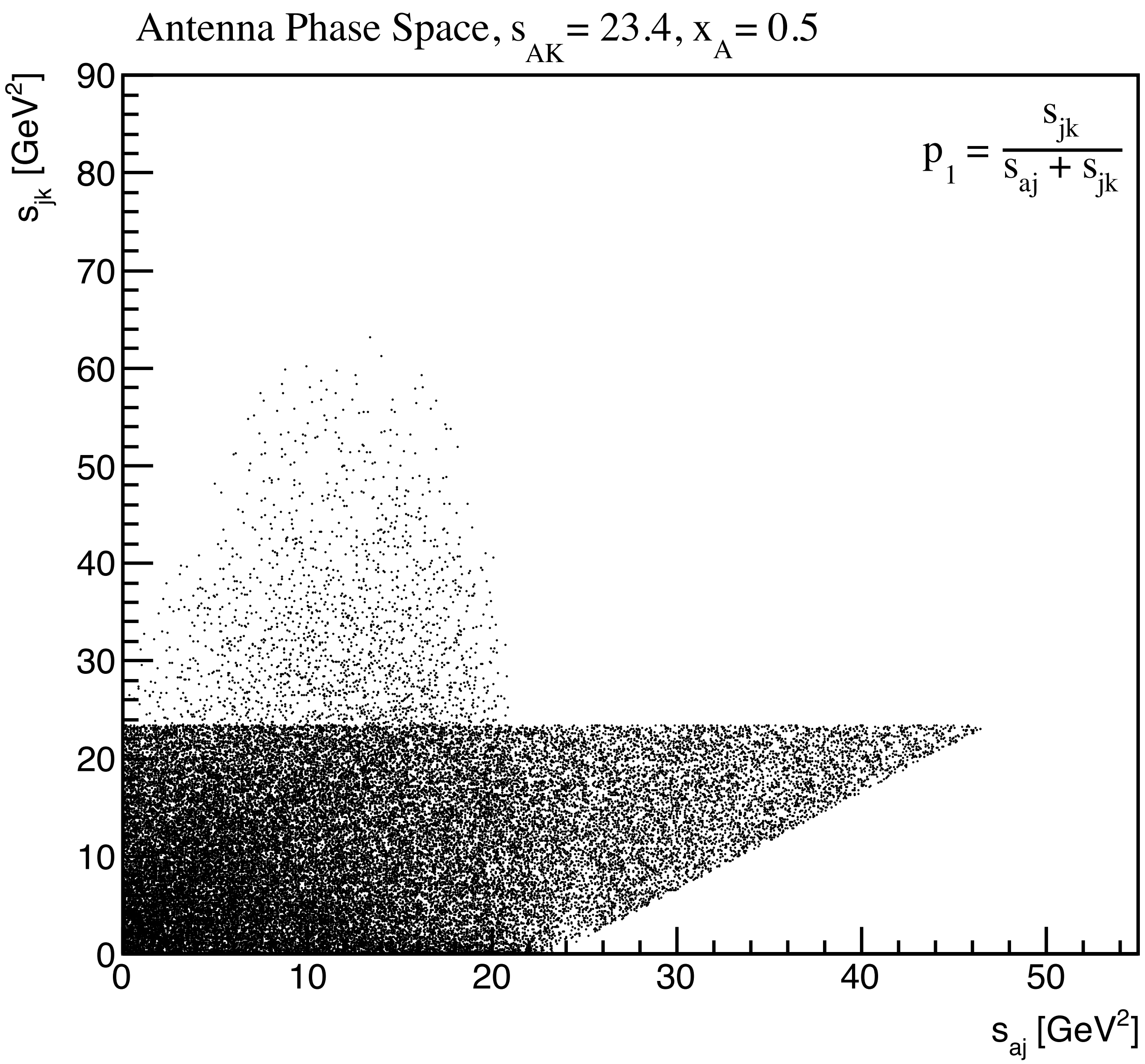
Correct Jacobian by veto with a factor

$$P_J = \frac{P_1 \left(\frac{x_{A1}}{x_{a1}}\right)^2 + (1 - P_1)\left(\frac{x_{A2}}{x_{a2}}\right)^2}{\left(\frac{x_{A1}}{x_{a1}}\right)^2}$$

Phase Space



Antenna Phase Space, $s_{AK} = 23.4$, $x_A = 0.5$
$p_1 = \frac{s_{aj}}{s_{aj} + s_{jk}}$
s_{jk} [GeV^2]
s_{aj} [GeV^2]

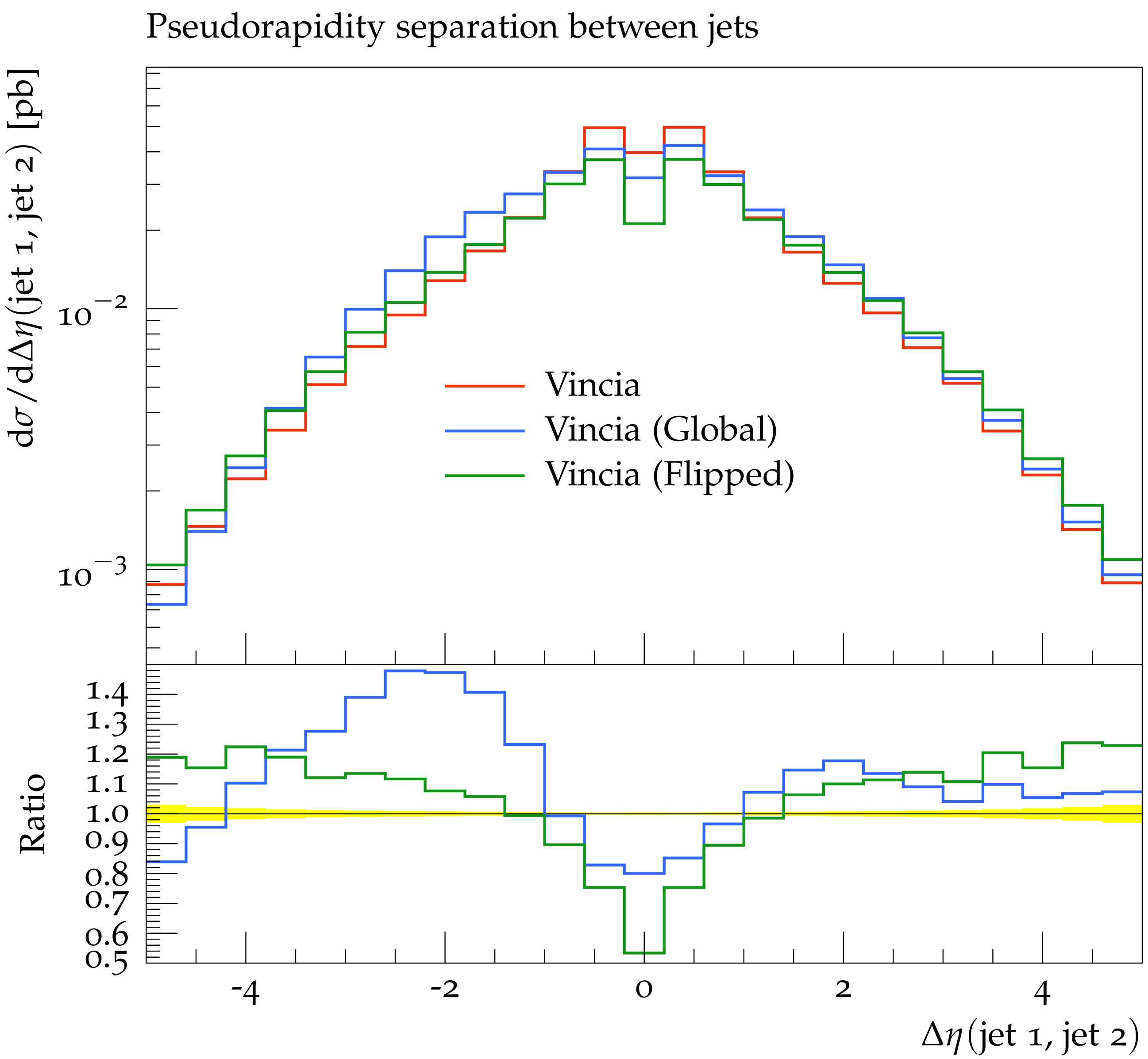


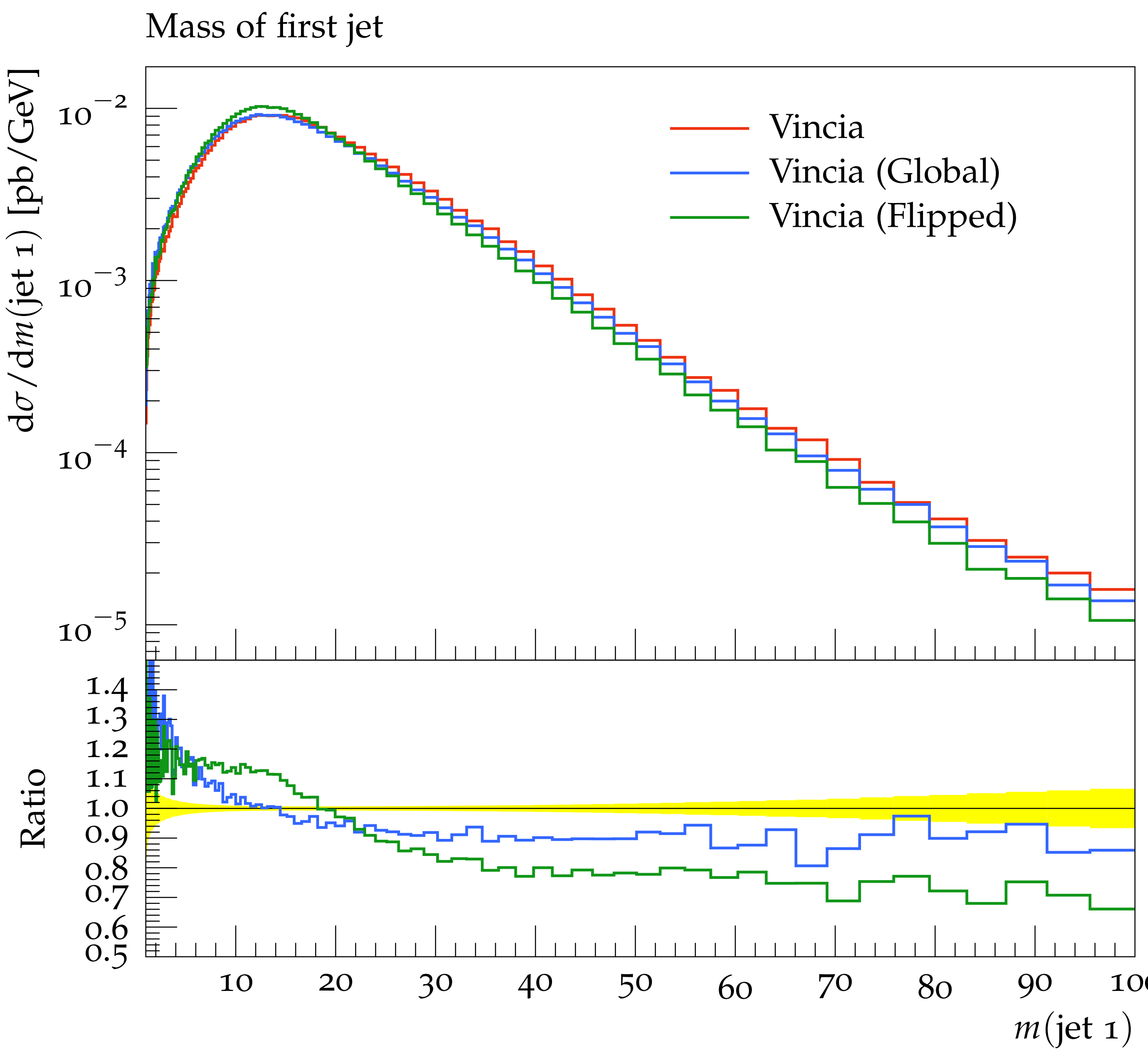
Antenna Phase Space, $s_{AK} = 23.4$, $x_A = 0.5$
$p_1 = \frac{s_{jk}}{s_{aj} + s_{jk}}$
s_{jk} [GeV^2]
s_{aj} [GeV^2]

Cabouat, Sjöstrand:1710.00391

Phase Space

$$q + \bar{q} \to \gamma^* / Z + g$$

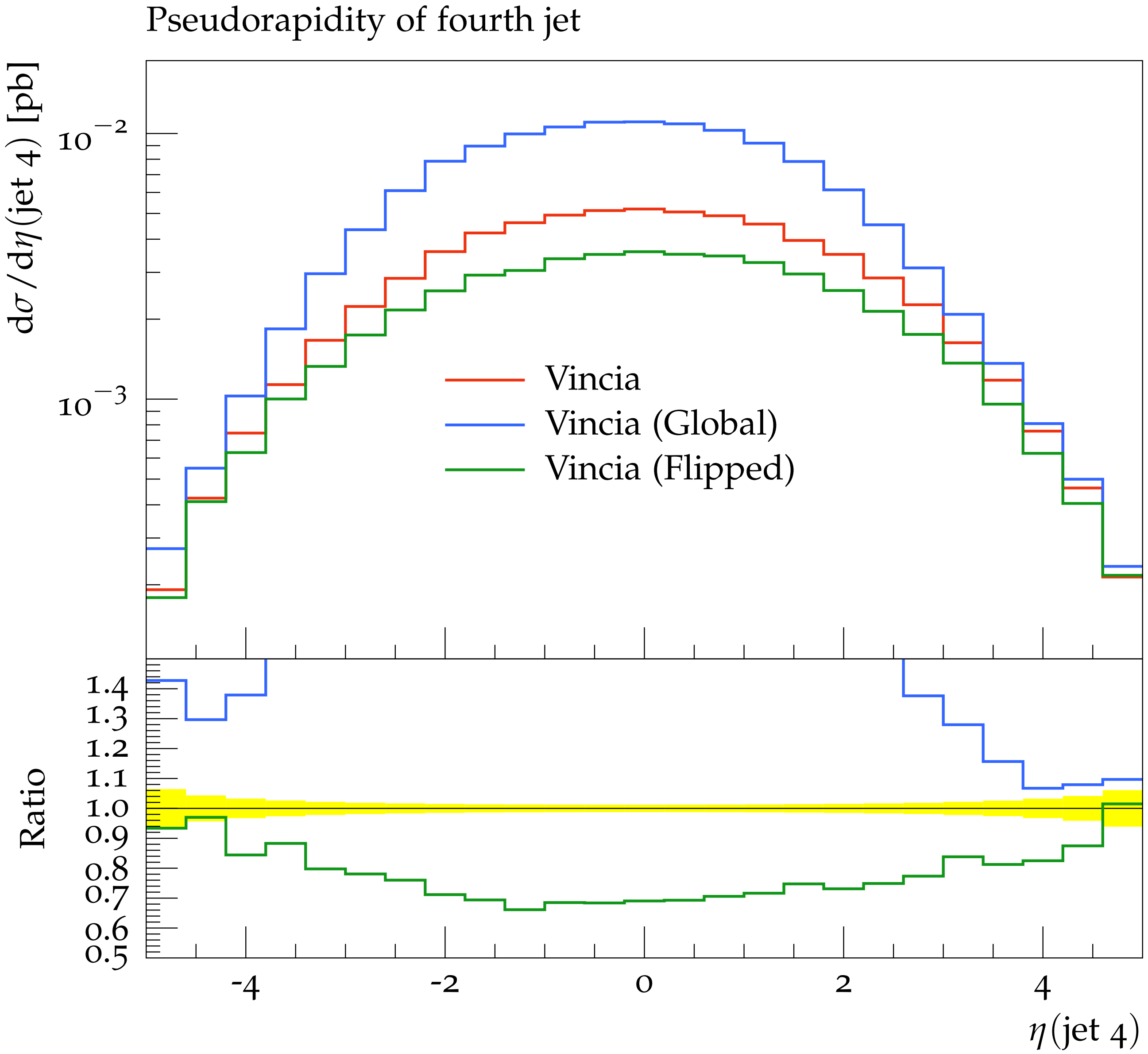


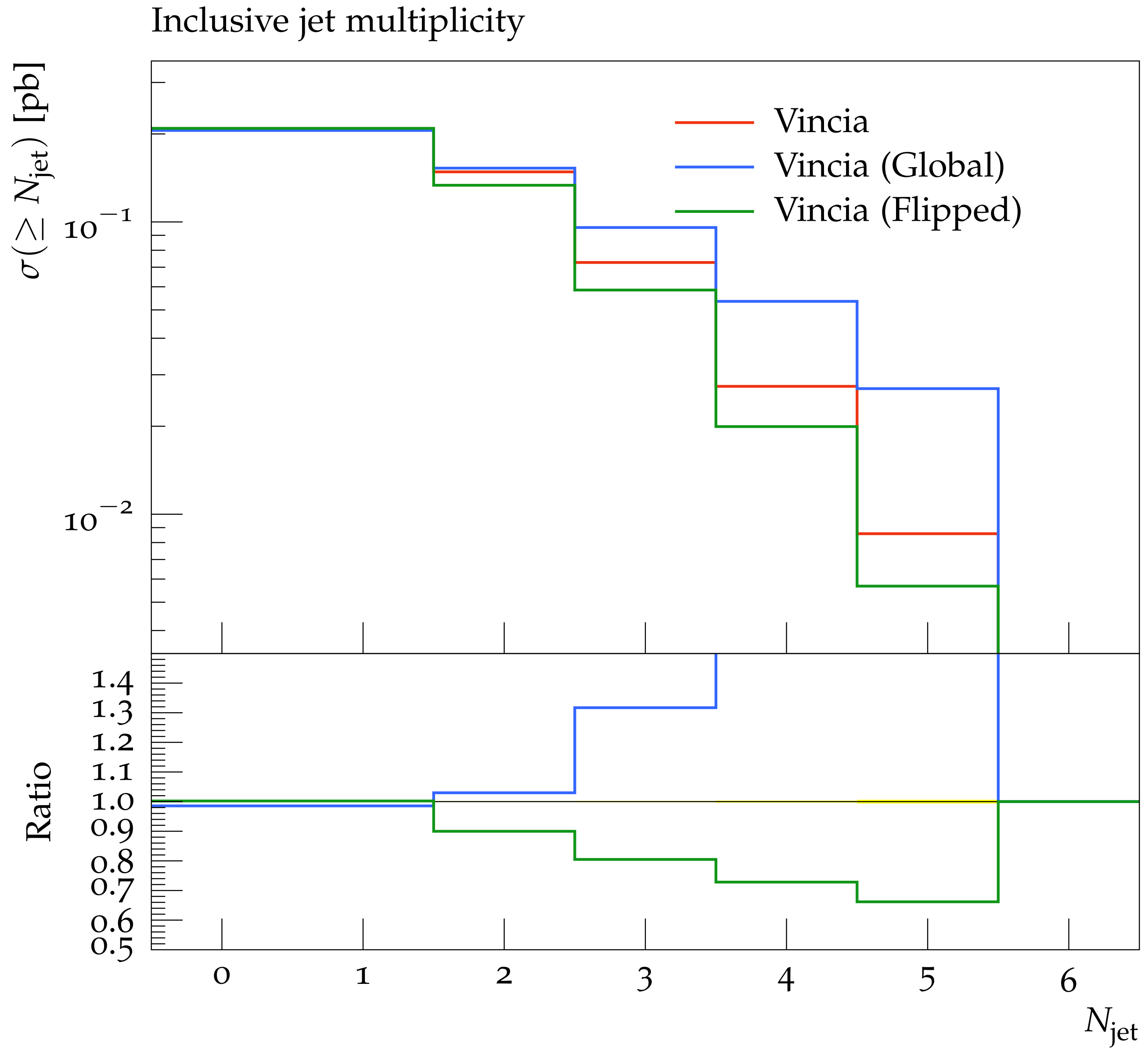


Cabouat, Sjöstrand:1710.00391

Phase Space

$$q + \bar{q} \rightarrow \gamma^* / Z + g$$





Conclusion