Electroweak Radiation in the Vincia Parton Shower

Rob Verheyen

In collaboration with Ronald Kleiss, Peter Skands, Helen Brooks

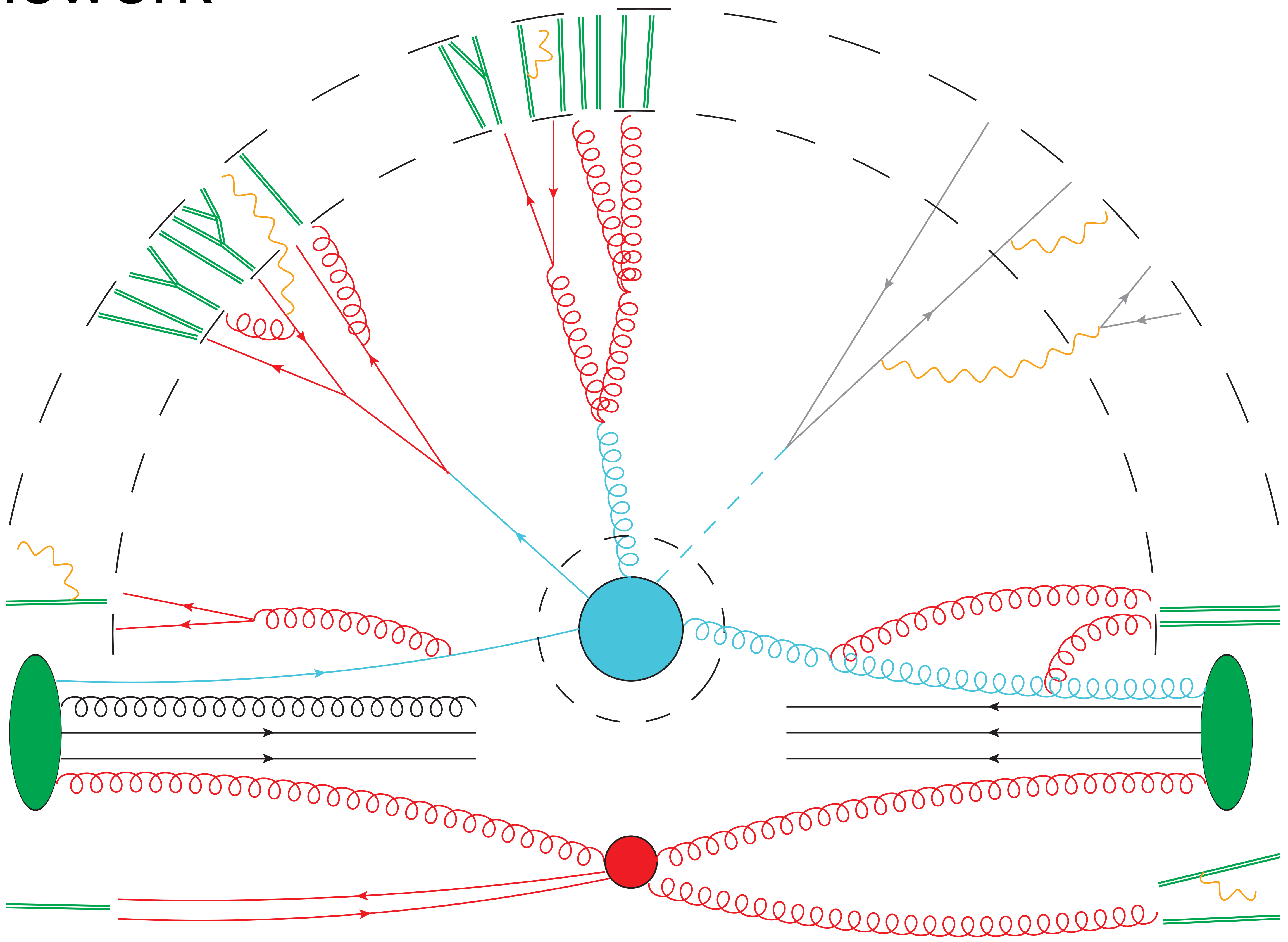
Overview

1. The Vincia Parton Shower
2. Electroweak Showering
3. Novel features in the Electroweak Sector

The Vincia parton shower

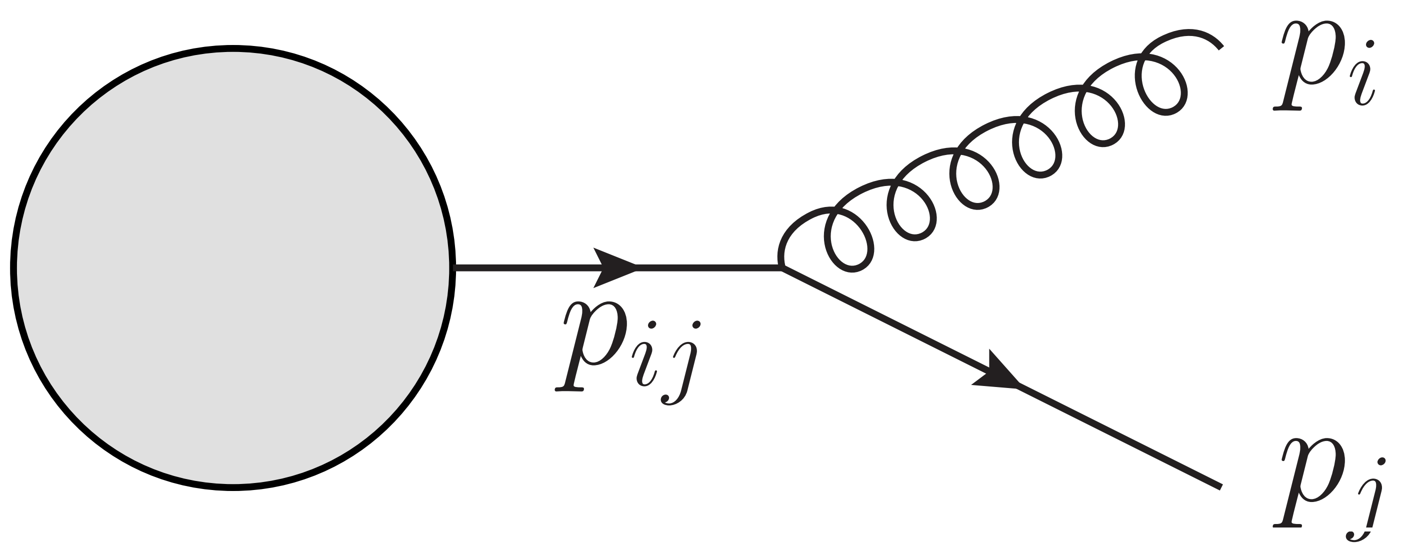
Parton Showers

- Essential part of Monte Carlo event generators
- Process-independent resummation framework
- Fully differential
- Interface hard scattering (high scale) to hadronization (low scale)
- Many types with many differences



Factorization

Based on factorisation properties of Matrix Element
in singular limits



1. Quasi-collinear limit

$$p_i \cdot p_j \approx m_i^2, m_j^2 \text{ and } E_i^2, E_j^2 \gg p_i \cdot p_j$$

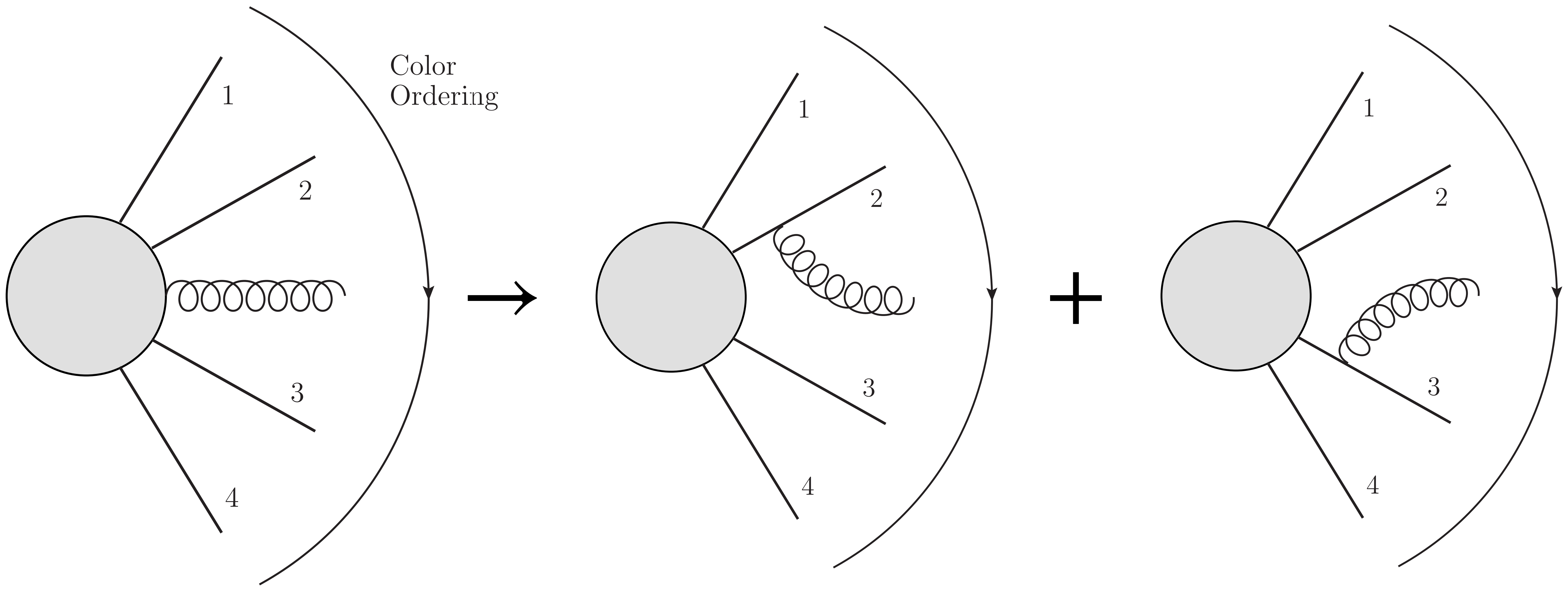
$$|M_{n+1}(.., p_i, p_j, ..)|^2 \to 8\pi\alpha_s \frac{1}{(p_i + p_j)^2} P_{(ij)\to ij}(z) |M_n(.., p_{ij}, ..)|^2$$

Factorization

Based on factorisation properties of Matrix Element
in singular limits

2. Soft limit

$E_j \approx m_j$ and $E_i, E_k \gg E_j$



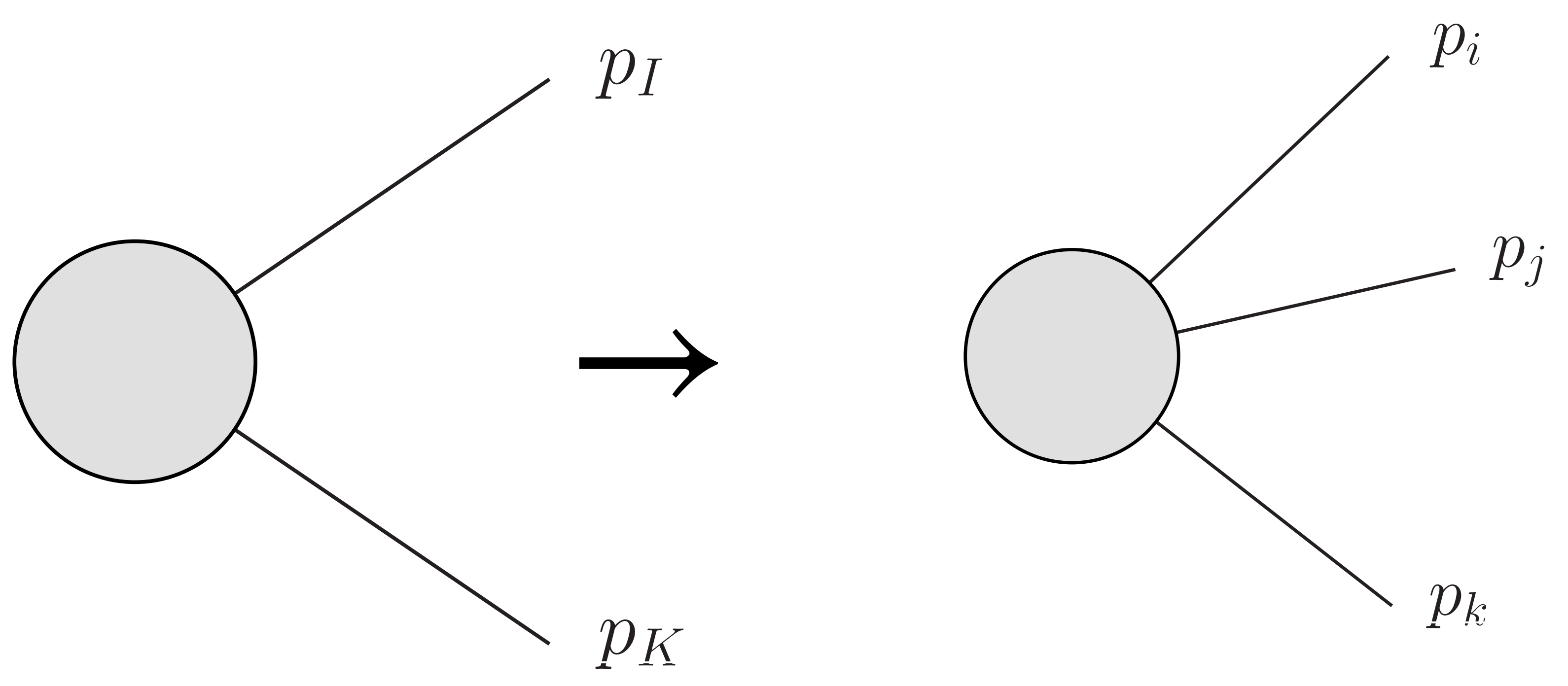
$$|M_{n+1}(.., p_i, p_j, p_k..)|^2 \to 4\pi\alpha_s C\left[2\frac{p_i{\cdot}p_k}{p_i{\cdot}p_j p_j{\cdot}p_k} - \frac{m_i^2}{(p_i{\cdot}p_j)^2} - \frac{m_k^2}{(p_j{\cdot}p_k)^2}\right]|M_n(.., p_i, p_k..)|^2$$

$$+\mathcal{O}(1/N_C^2)$$

Main Ingredients

1. Phase space factorisation → Comes with a **kinematic map**

$$d\Phi_{n+1} = d\Phi_n \times d\Phi_{\text{ps}}$$



1. Momentum conservation
2. IR safety

2. Ordering scale

$$p_\perp^2(\Phi_{\text{ps}})$$

3. Branching kernel

$$|M_{n+1}(\Phi_{n+1})|^2 \approx \sum_i B_i(\Phi_{\text{ps}}) \times |M_n(\Phi_n)|^2$$

Parton Showers

Branching kernel (real corrections)

↓

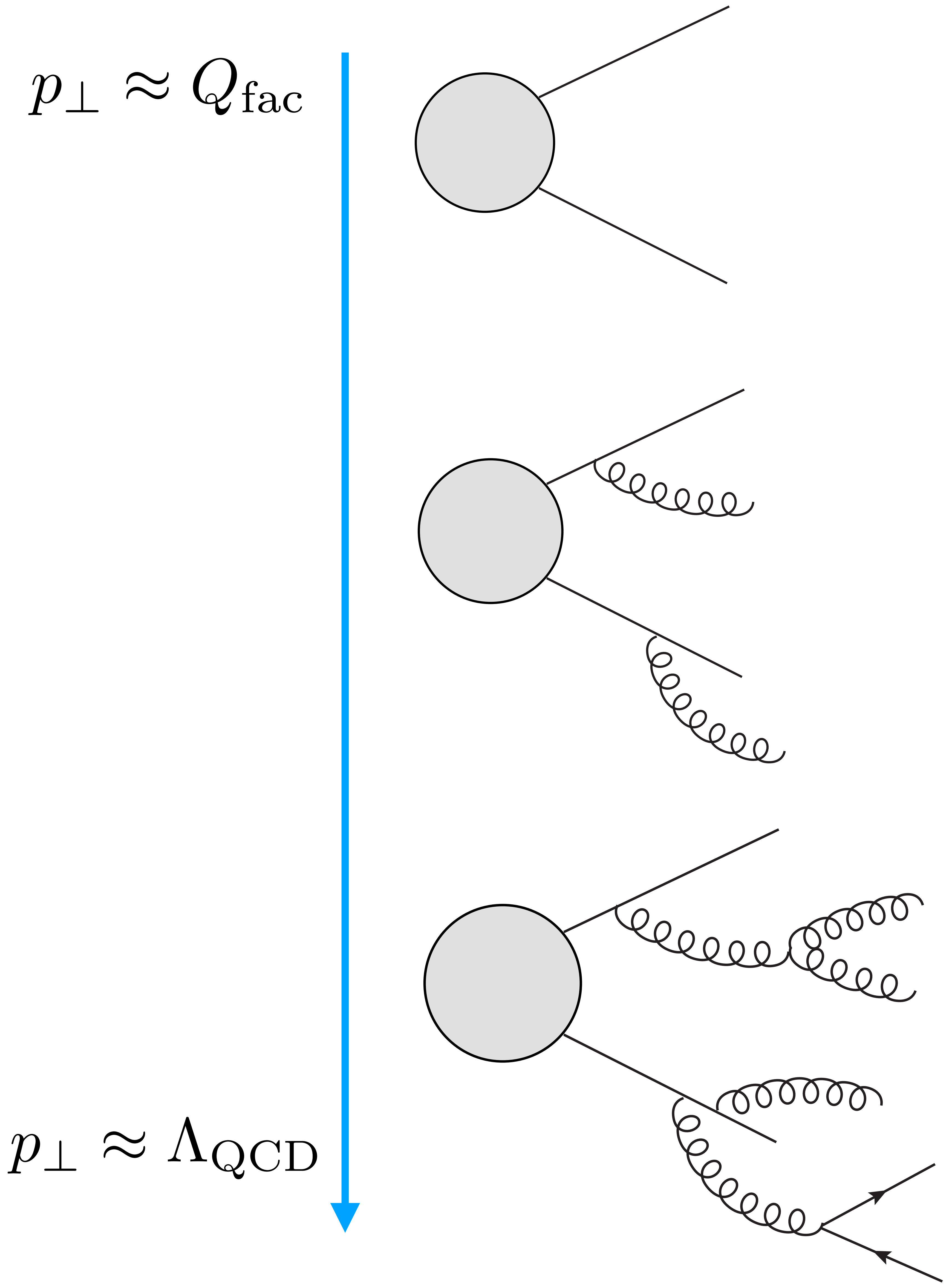
$$P_i(\Phi_{\text{ps},i}) = B(\Phi_{\text{ps},i})\Theta(p^2_{\perp,i} < p^2_{\perp,i-1}) \times \Delta(p^2_{\perp,i-1}, p^2_{\perp,i})$$

↑

Sudakov factor (virtual corrections)

$$\Delta(p^2_{\perp,i-1}, p^2_{\perp,i}) = \exp\left(-\int_{p^2_{\perp,i}}^{p^2_{\perp,i-1}} d\Phi_{\text{ps}} B(\Phi_{\text{ps}})\right)$$

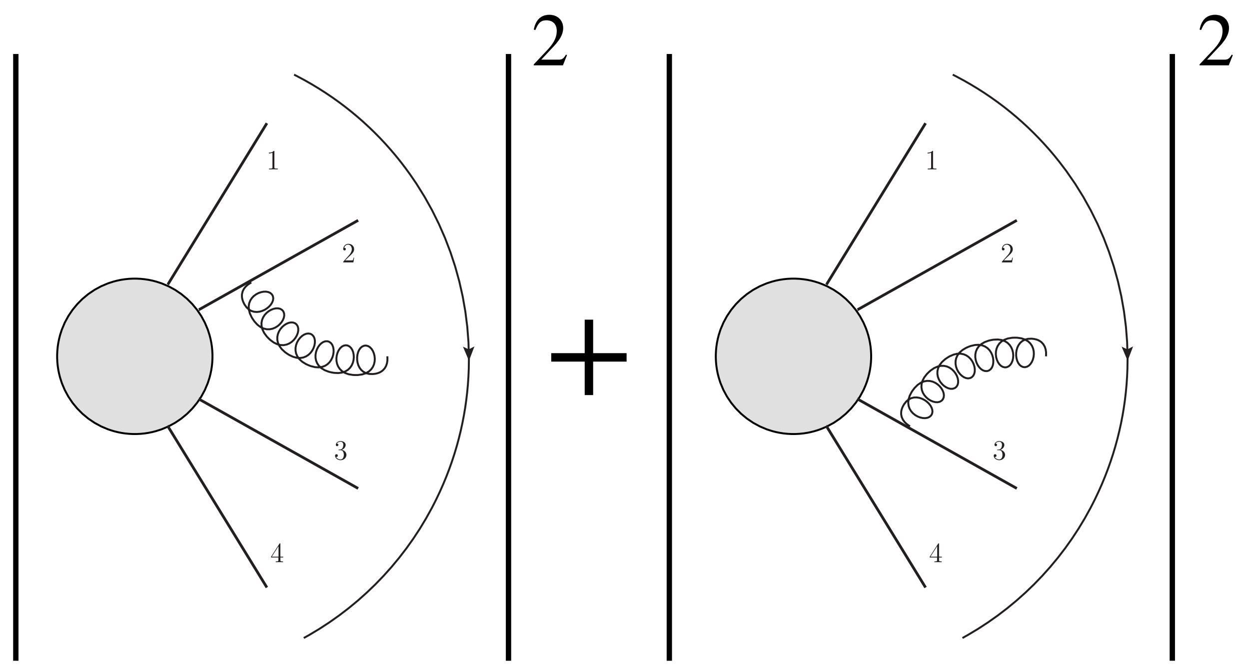
Parton shower is *unitary*:
cancellation of real and virtual corrections
$\longrightarrow \sigma_{\text{inc}}$ unaltered



Types of Showers

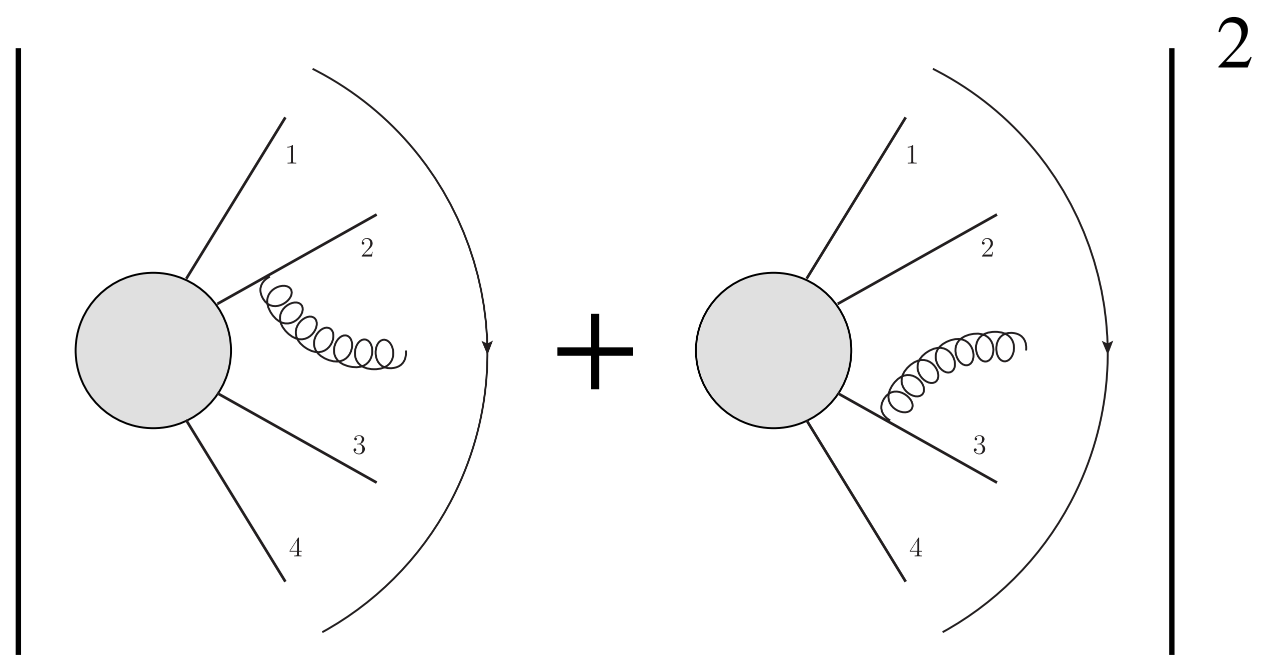
Dipole

- Pythia
- Dire
- Sherpa
- Herwig



Antenna

- Vincia
- Ariadne



- Collinear limit automatic
- Coherence through angular ordering or partial fractioning
- Distinct difference emitter vs. spectator

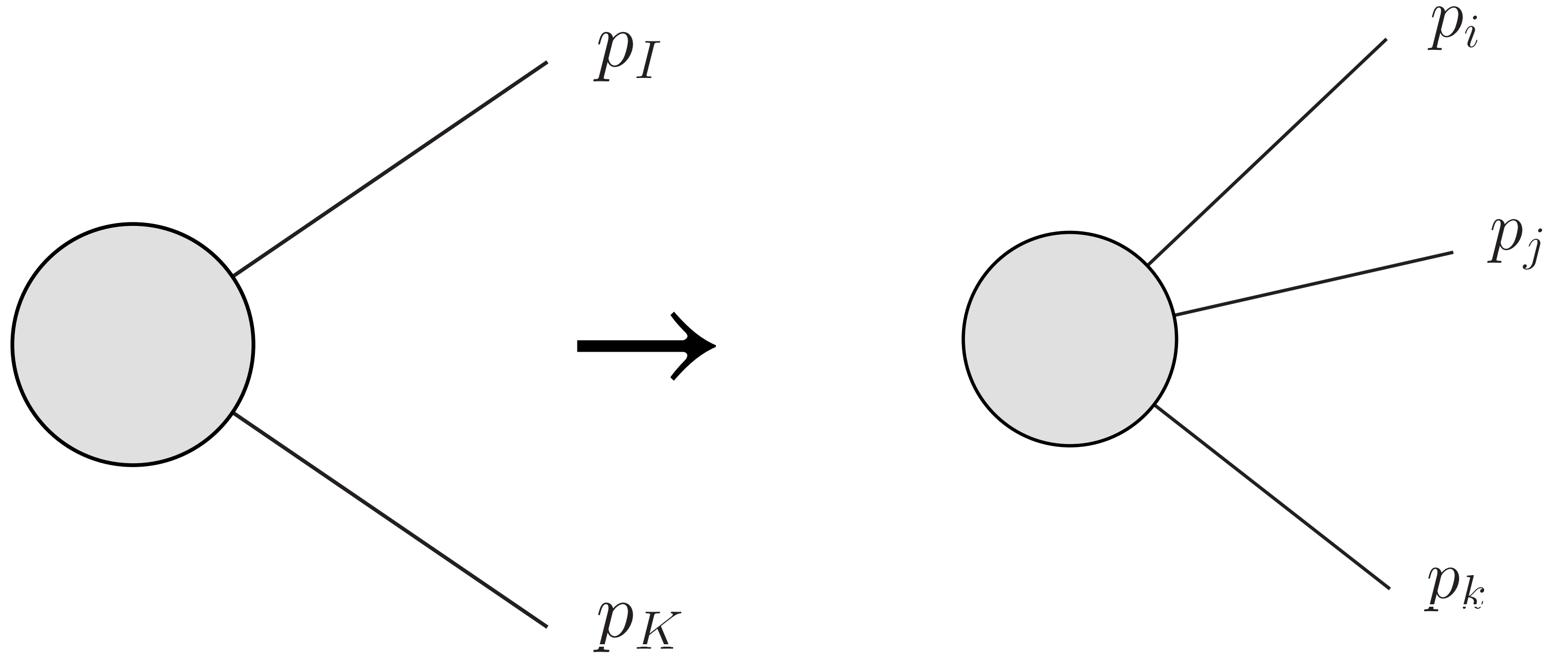
- Collinear limit automatic (some subtleties)
- Coherence automatic
- Emitters treated on equal footing

Vincia: Three ingredients

$$s_{ab} = 2p_a \cdot p_b$$
$$m_{ab}^2 = (p_a + p_b)^2$$

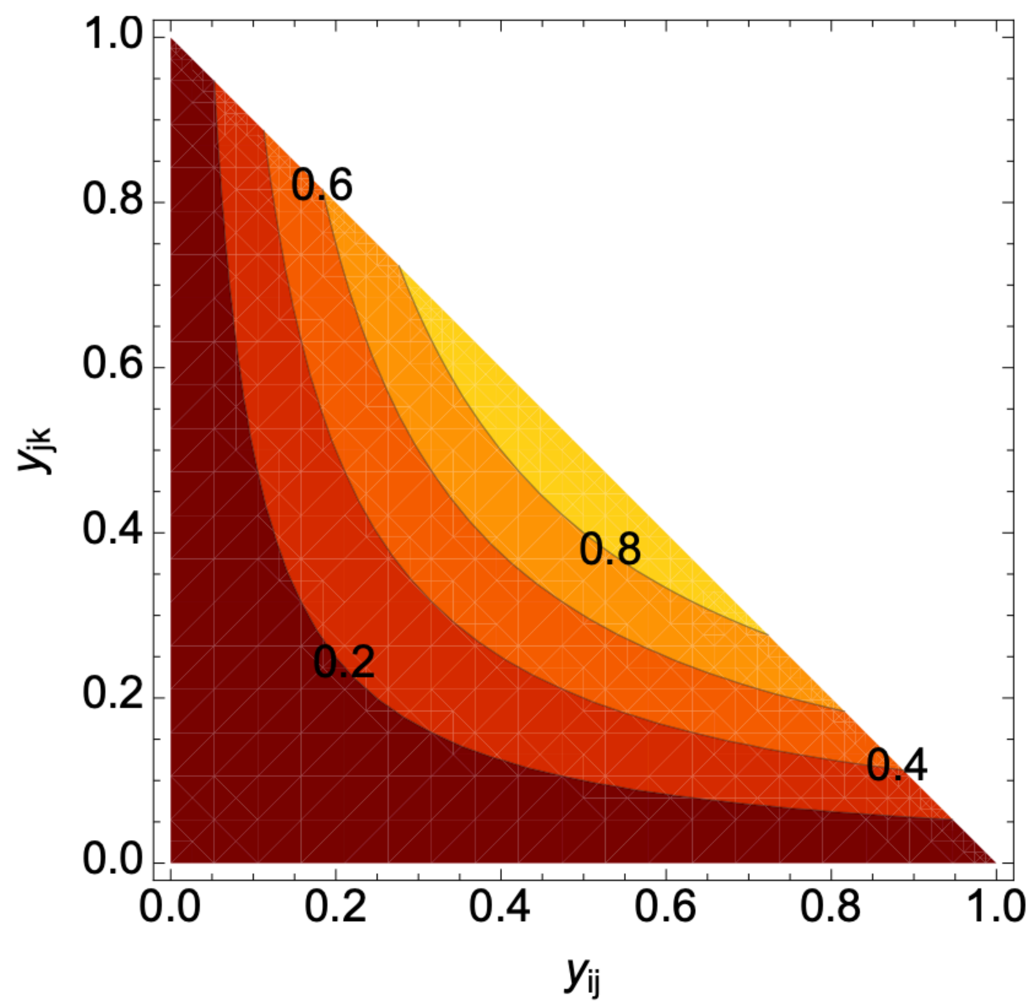
1. Phase space factorisation

$$d\Phi_{\rm ps} = \frac{1}{16\pi^2}\lambda^{\frac{1}{2}}(m_{IK}^2, m_I^2, m_K^2)\, ds_{ij}\, ds_{jk}\, \frac{d\varphi}{2\pi}$$



2. Ordering scale: Ariadne $p_\perp^2$

$$p_\perp^2 = \frac{s_{ij}s_{jk}}{s_{IK}}$$



3. Branching kernel: Antenna functions

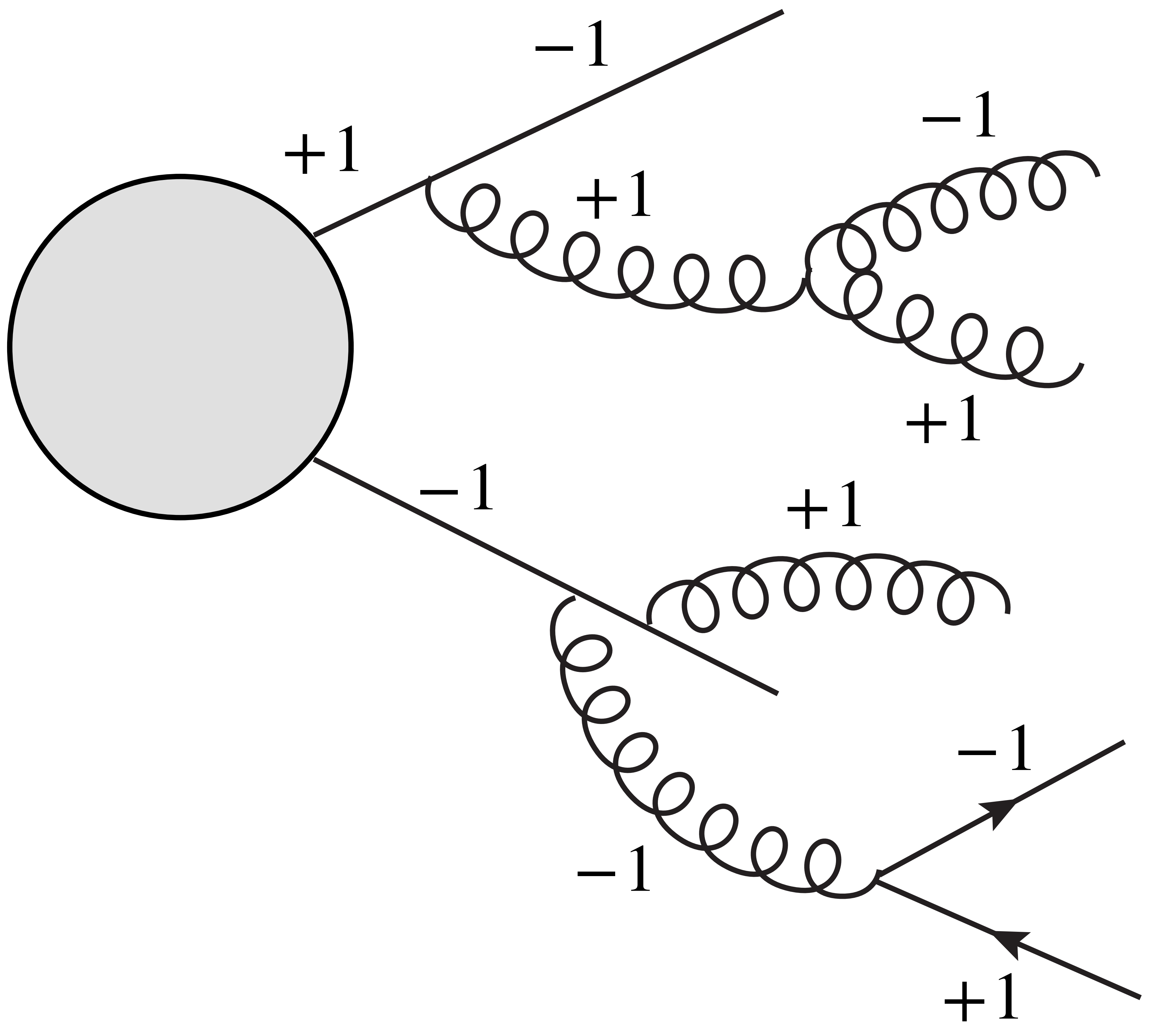
$$a_{q\bar{q}}(s_{ij}, s_{jk}) = 4\pi\alpha_s C_F\left(2\frac{s_{ik}}{s_{ij}s_{jk}} - 2\frac{m_i^2}{s_{ij}^2} - 2\frac{m_k^2}{s_{jk}^2} + \frac{1}{s_{IK}}\left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}}\right)\right)$$

Vincia: Helicity showers

Evolution of intermediate helicity states

⟶ Separate antenna function for every configuration

Very important in EW sector

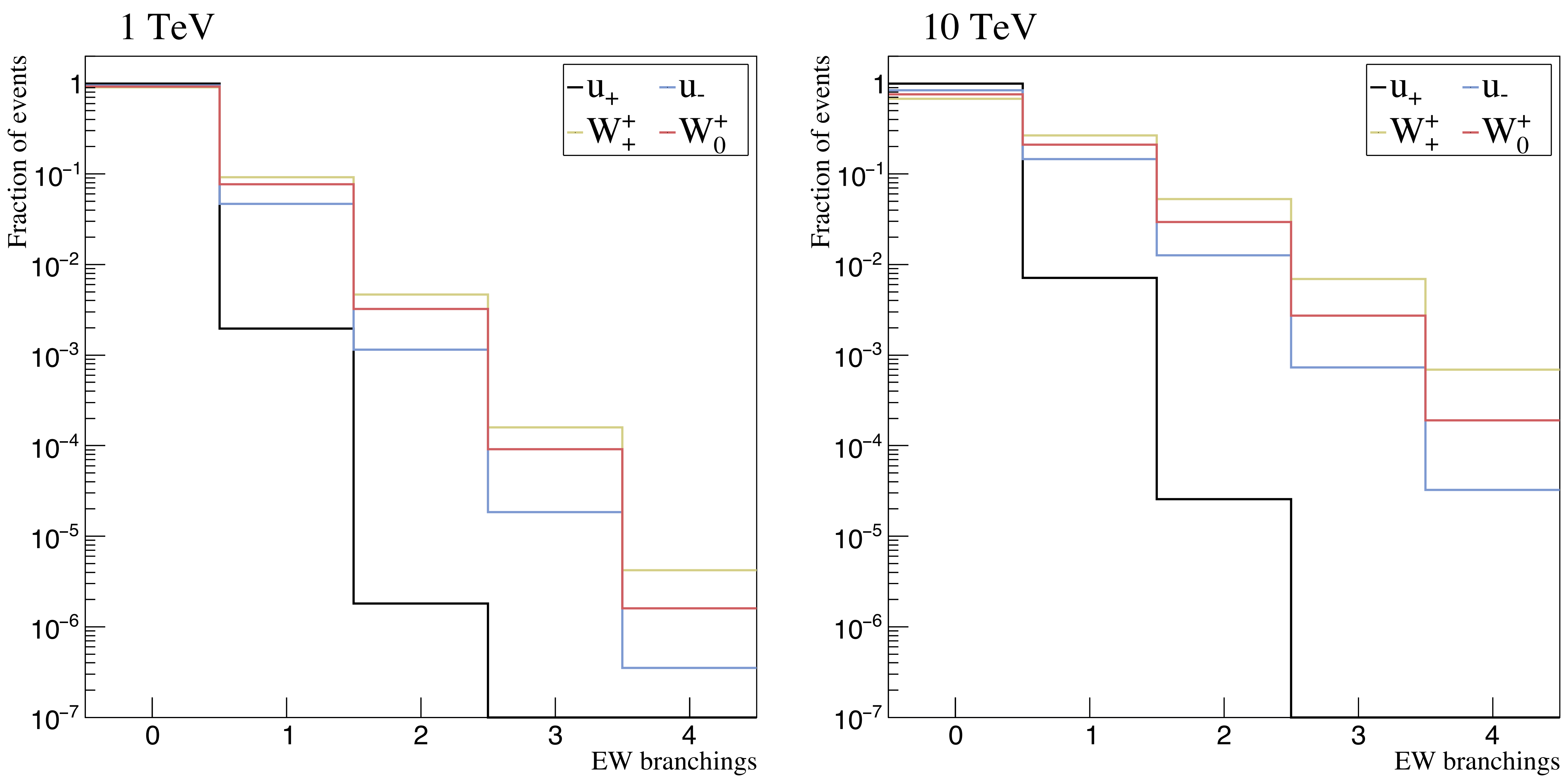


Note: Not full spin correlations, only diagonal contributions

Electroweak Showering

Why EW Showers?

- **EW gauge bosons, tops, Higgs part of jets**
- **Universal incorporation of EW virtual corrections** $\propto \log\left(\frac{\hat{s}}{Q^2_{EW}}\right)$



Just starting to become relevant

- (HL)-LHC — ATLAS 1609.07045
- Future colliders

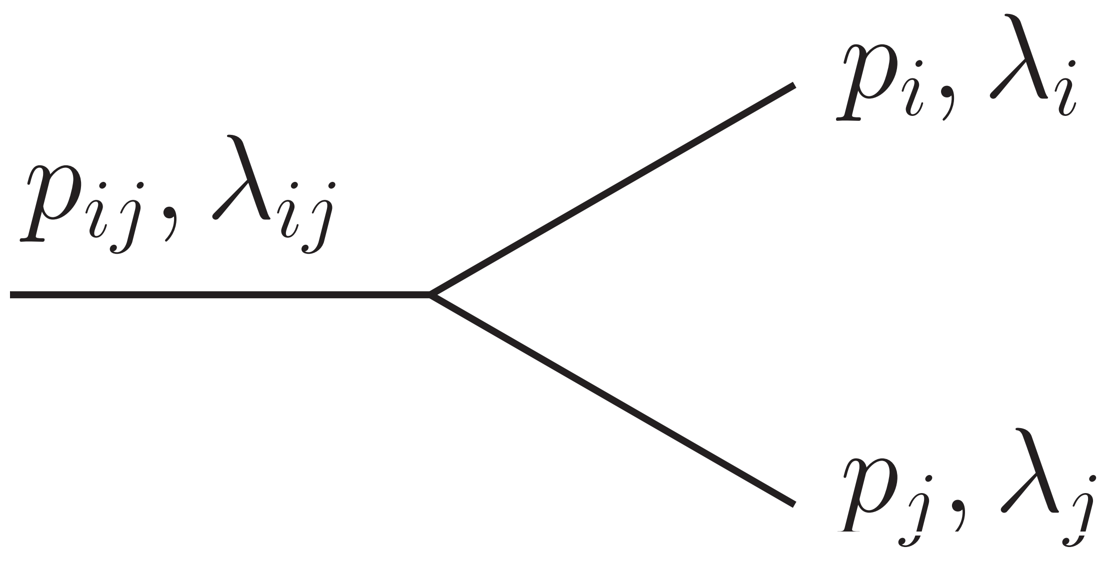
Existing implementations

- Only vector boson emissions — Christiansen, Sjostrand arXiv:1401.5238; Krauss, Petrov, Schoenherr, Spannowsky arXiv:1403.4788
- Full-fledged EW shower — Chen, Han, Tweedie arXiv:1611.00788

Electroweak Branching Kernels

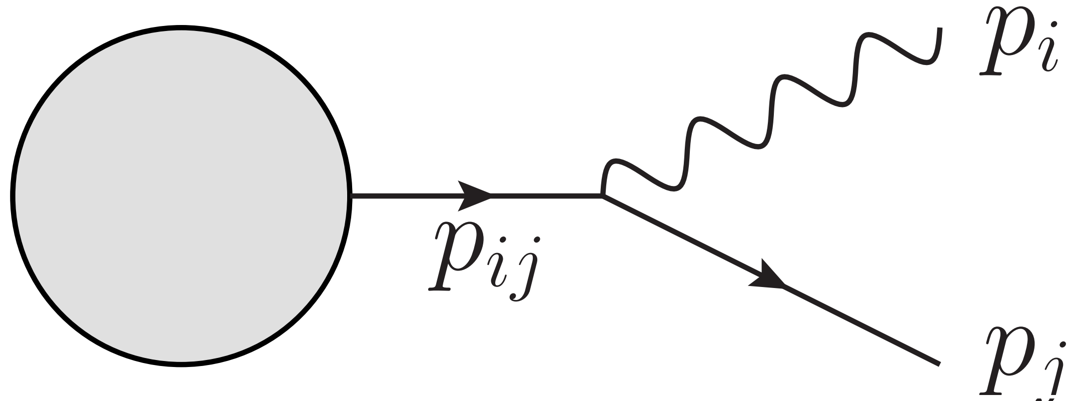
Use spinor-helicity formalism

$$M_{\lambda_{ij},\lambda_i,\lambda_j}(p_i,p_j) =$$



Transform to Vincia phase space

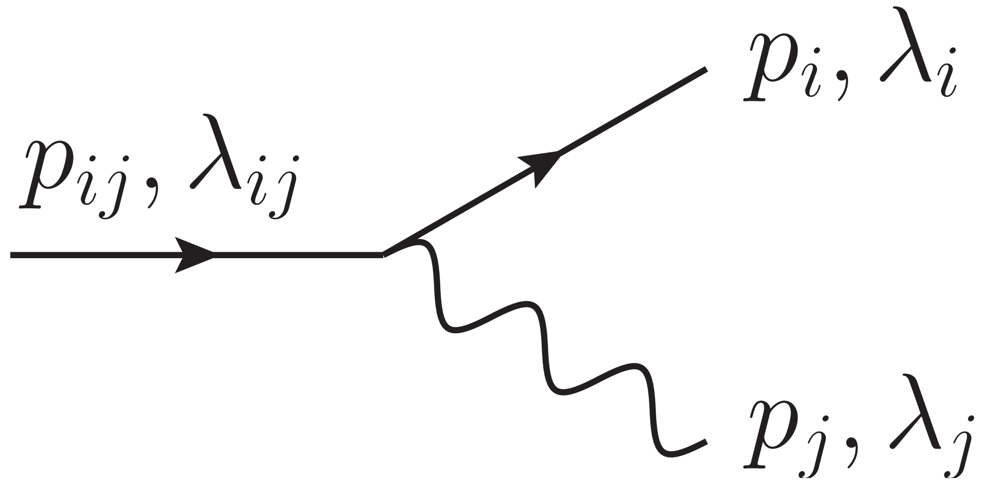
$$a_{\lambda_{ij},\lambda_i,\lambda_j}(s_{ij}, s_{jk}) = \left[\left| \frac{1}{Q^2} M_{\lambda_{ij},\lambda_i,\lambda_j}(p_i,p_j) \right|^2 \right]^{z \to x_i}_{(1-z) \to x_j}$$



$$x_i = \frac{s_{ij} + s_{ik} + m_i^2}{m_{IK}^2} \qquad x_j = \frac{s_{ij} + s_{jk} + m_j^2}{m_{IK}^2}$$

$$Q^2 = s_{ij} + m_i^2 + m_j^2 - m_{ij}^2$$

Longitudinal Polarisations



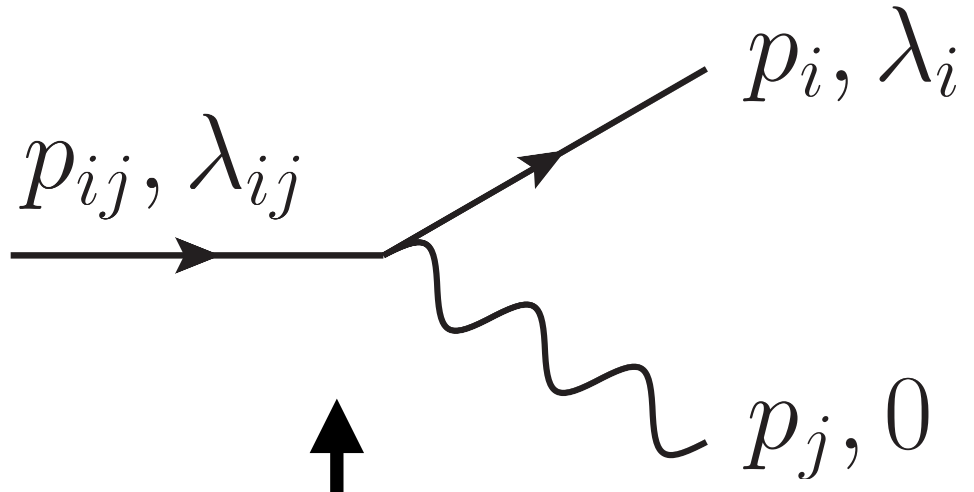
$$M_{\lambda_{ij},\lambda_i,\lambda_j}(p_i, p_j) = \bar{u}_{\lambda_i}(p_i)(v + a\gamma^5)\not{\epsilon}_{\lambda_j}(p_j)u_{\lambda_{ij}}(p_{ij})$$

1. Insert spinor representations
2. Consider longitudinal polarisation
3. Do some Dirac algebra

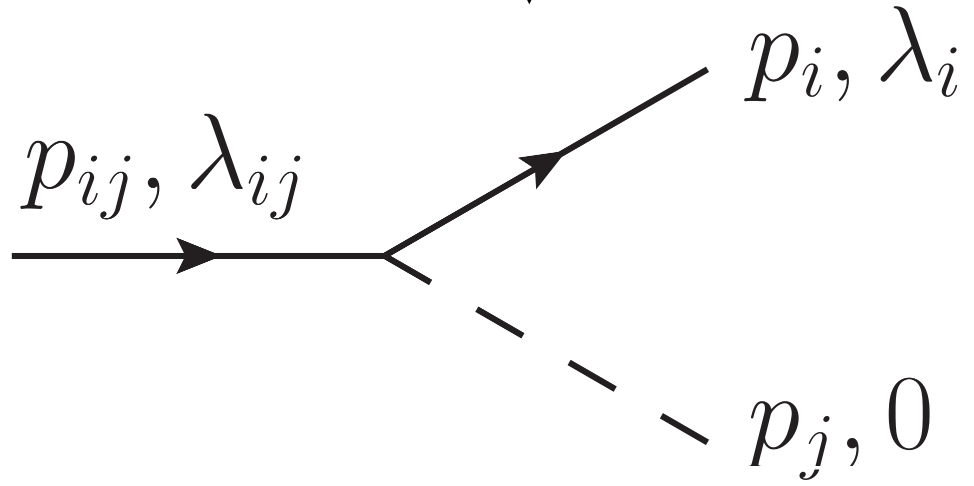
$$M_{+,+,0}(p_i, p_j) \propto \frac{1}{m_j}\left((Q^2 + m_{ij}^2)\not{p}_{ij} - m_i^2\not{p}_{ij}\right)$$

Q^2 drops out → Unitarity violation → ???

Goldstone Bosons



$$\epsilon_0^\mu(p) = \frac{1}{m}\left(p^\mu - \frac{m^2}{p\cdot k}k^\mu\right)$$



Goldstone piece actually couples to Yukawa

Possible to solve with Goldstone equivalence and suitable gauge choice

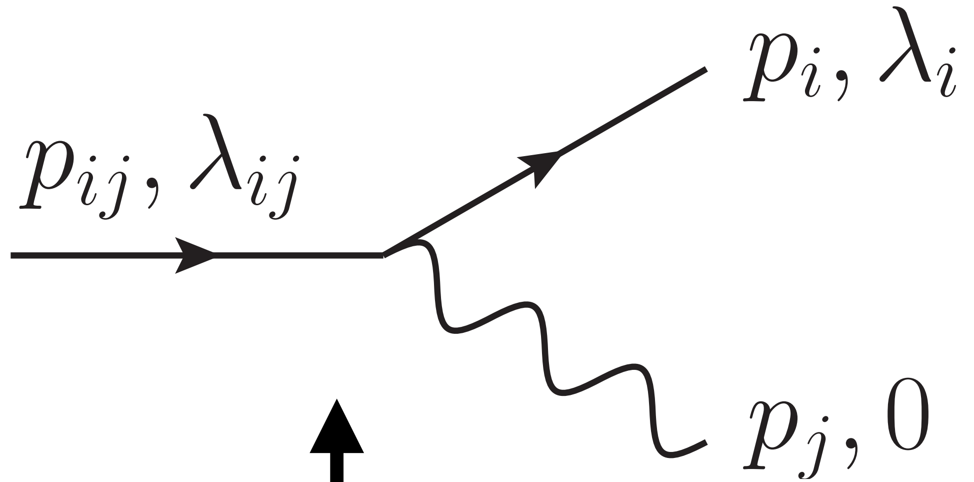
Spinor helicity formalism enables much simpler solution:

Yukawa couplings

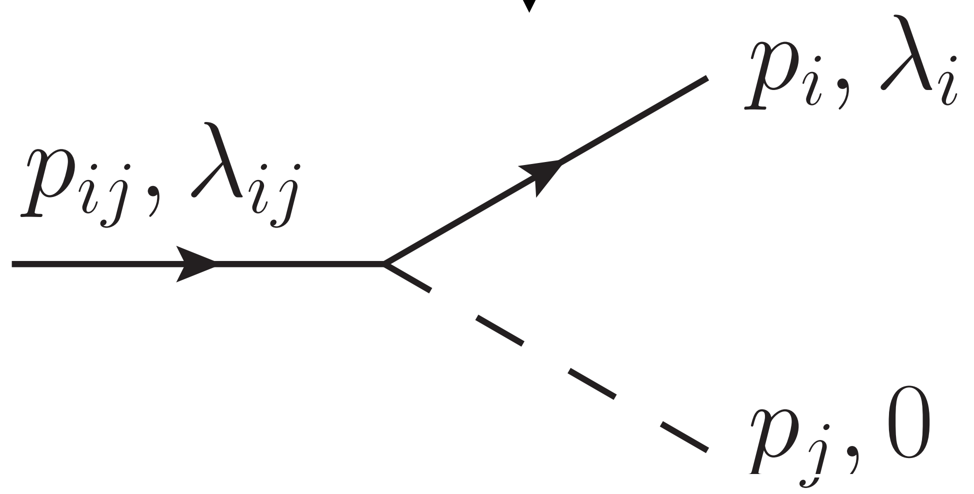
$$\frac{1}{m_j}\left((Q^2 + m_{ij}^2)\not{p}_i - m_i^2\not{p}_{ij}\right)$$

Off-shellness

Goldstone Bosons



$$\epsilon_0^\mu(p) = \frac{1}{m}\left(p^\mu - \frac{m^2}{p\cdot k}k^\mu\right)$$



Goldstone piece actually couples to Yukawa

Possible to solve with Goldstone equivalence and suitable gauge choice

Spinor helicity formalism enables much simpler solution:

Yukawa couplings

$$\frac{1}{m_j}\left((\cancel{Q^2} + m_{ij}^2)\not{p}_i - m_i^2\not{p}_{ij}\right)$$

Off-shellness

Collinear Limits

λ_I	λ_i	λ_j	$V \to f\bar{f}'$
λ	λ	$-\lambda$	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,z$
λ	$-\lambda$	λ	$\sqrt{2}\lambda(v+\lambda a)\sqrt{\tilde{Q}^2}\,(1-z)$
λ	λ	λ	$\sqrt{2}\lambda\left[m_i(v+\lambda a)\sqrt{\frac{1-z}{z}}+m_j(v-\lambda a)\sqrt{\frac{z}{1-z}}\right]$
λ	$-\lambda$	$-\lambda$	0
0	λ	λ	$\sqrt{\tilde{Q}^2}\left[\frac{m_i}{m_{ij}}(v+\lambda a)+\frac{m_j}{m_{ij}}(v-\lambda a)\right]$
0	λ	$-\lambda$	$(v-\lambda a)\left[2m_{ij}\sqrt{z(1-z)}-\frac{m_i^2}{m_{ij}}\sqrt{\frac{1-z}{z}} - \frac{m_j^2}{m_{ij}}\sqrt{\frac{z}{1-z}}\right]+(v+\lambda a)\frac{m_i m_j}{m_{ij}}\frac{1}{\sqrt{z(1-z)}}$

λ_{ij}	λ_i	λ_j	$f \to f'V$ and $\bar{f} \to \bar{f}'V$
λ	λ	λ	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,\frac{1}{\sqrt{1-z}}$
λ	λ	$-\lambda$	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,\frac{z}{\sqrt{1-z}}$
λ	$-\lambda$	λ	$\sqrt{2}\lambda\left[m_{ij}(v-\lambda a)\sqrt{z}-m_i(v+\lambda a)\frac{1}{\sqrt{z}}\right]$
λ	$-\lambda$	$-\lambda$	0
λ	λ	0	$(v-\lambda a)\left[\frac{m_{ij}^2}{m_j}\sqrt{z}-\frac{m_i^2}{m_j}\frac{1}{\sqrt{z}}-2m_j\frac{\sqrt{z}}{1-z}\right]+(v+\lambda a)\frac{m_i m_{ij}}{m_j}\frac{1-z}{\sqrt{z}}$
λ	$-\lambda$	0	$\sqrt{\tilde{Q}^2}\sqrt{1-z}\left[\frac{m_i}{m_j}(v-\lambda a)-\frac{m_{ij}}{m_j}(v+\lambda a)\right]$

λ_I	λ_i	$(f \to fh$ and $\bar{f} \to \bar{f}h) \times \frac{e}{2s_w}\frac{m_f}{m_w}$
λ	λ	$m_f\left[\sqrt{z}+\frac{1}{\sqrt{z}}\right]$
λ	$-\lambda$	$\sqrt{1-z}\sqrt{\tilde{Q}^2}$

λ_I	λ_i	$V \to Vh \times g_h$
λ	λ	-1
λ	$-\lambda$	0
0	λ	$\frac{1}{m_{ij}}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{z(1-z)}$
λ	0	$\frac{1}{m_i}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
0	0	$\frac{1}{2}\frac{m_j^2}{m_i^2}+\frac{1-z}{z}+z$

λ_i	λ_i	$h \to VV \times g_V$
λ	λ	0
λ	$-\lambda$	-1
0	λ	$\frac{1}{m_i}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
λ	0	$\frac{1}{m_j}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{z}{1-z}}$
0	0	$\frac{1}{2}\frac{m_{ij}^2}{m_i^2}-1-\frac{1-z}{z}-\frac{z}{1-z}$

λ_i	λ_j	$h \to f\bar{f} \times \frac{e}{2s_w}\frac{m_f}{m_w}$
λ	λ	$\sqrt{\tilde{Q}^2}$
λ	$-\lambda$	$m_f\left[\sqrt{\frac{1-z}{z}}-\sqrt{\frac{z}{1-z}}\right]$

λ_I	λ_i	λ_j	$V \to V'V'' \times g_V$
λ	λ	λ	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}\sqrt{\frac{1}{z(1-z)}}$
λ	λ	$-\lambda$	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}z\sqrt{\frac{z}{1-z}}$
λ	$-\lambda$	λ	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}(1-z)\sqrt{\frac{1-z}{z}}$
λ	$-\lambda$	$-\lambda$	0
0	λ	λ	0
0	λ	$-\lambda$	$m_{ij}(2z-1)+\frac{m_j^2}{m_{ij}}-\frac{m_i^2}{m_{ij}}$
λ	0	λ	$m_i\left(1+2\frac{1-z}{z}\right)+\frac{m_j^2}{m_i}-\frac{m_{ij}^2}{m_i}$
λ	0	$-\lambda$	0
λ	λ	0	$m_j\left(1+2\frac{z}{1-z}\right)+\frac{m_i^2}{m_j}-\frac{m_{ij}^2}{m_j}$
λ	$-\lambda$	0	0
λ	0	0	$\frac{\lambda}{\sqrt{2}}\frac{m_i^2+m_j^2-m_{ij}^2}{m_i m_j}\sqrt{\tilde{Q}^2}\sqrt{z(1-z)}$
0	λ	0	$\frac{\lambda}{\sqrt{2}}\frac{m_{ij}^2+m_j^2-m_i^2}{m_{ij} m_j}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
0	0	λ	$\frac{\lambda}{\sqrt{2}}\frac{m_{ij}^2+m_i^2-m_j^2}{m_{ij} m_i}\sqrt{\tilde{Q}^2}\sqrt{\frac{z}{1-z}}$
0	0	0	$\frac{1}{2}\frac{m_{ij}^3}{m_i m_j}(2z-1)-\frac{m_i^3}{m_{ij}m_j}\left(\frac{1}{2}+\frac{1-z}{z}\right)+\frac{m_j^3}{m_{ij}m_i}\left(\frac{1}{2}+\frac{z}{1-z}\right)+\frac{m_i m_j}{m_{ij}}\left(\frac{1-z}{z}-\frac{z}{1-z}\right)+\frac{m_{ij}m_i}{m_j}(1-z)\left(2+\frac{1-z}{z}\right)-\frac{m_{ij}m_j}{m_i}z\left(2+\frac{z}{1-z}\right)$

Collinear Limits

λ_{ij}	λ_i	λ_j	$f \to f'V$ and $\bar{f} \to \bar{f}'V$
λ	λ	λ	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,\frac{1}{\sqrt{1-z}}$
λ	λ	$-\lambda$	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,\frac{z}{\sqrt{1-z}}$
λ	$-\lambda$	λ	$\sqrt{2}\lambda\left[m_{ij}(v-\lambda a)\sqrt{z} - m_i(v+\lambda a)\frac{1}{\sqrt{z}}\right]$
λ	$-\lambda$	$-\lambda$	0
λ	λ	0	$(v-\lambda a)\left[\frac{m_{ij}^2}{m_j}\sqrt{z} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{z}} - 2m_j\frac{\sqrt{z}}{1-z}\right] + (v+\lambda a)\frac{m_i m_{ij}}{m_j}\frac{1-z}{\sqrt{z}}$
λ	$-\lambda$	0	$\sqrt{\tilde{Q}^2}\sqrt{1-z}\left[\frac{m_i}{m_j}(v-\lambda a) - \frac{m_{ij}}{m_j}(v+\lambda a)\right]$

First two rows (λ, λ, λ and λ, λ, −λ): $P(z) \propto \frac{\tilde{Q}^2}{Q^4}\frac{1+z^2}{1-z}$

Rows (λ, −λ, λ) and (λ, λ, 0): $P(z) \propto \frac{m^2}{Q^4}$

Row (λ, −λ, 0): $P(z) \propto \frac{\tilde{Q}^2}{Q^4}(1-z)$

$$\tilde{Q}^2 = Q^2 + m_{ij}^2 - \frac{m_i^2}{z} - \frac{m_j^2}{1-z}$$

The Electroweak Shower

- As similar as possible to the QCD shower
- $\mathcal{O}(1000)$ branchings (all FSR + ffV ISR)
- Ordering scale $p_\perp^2 = \dfrac{(s_{ij} + m_i^2 + m_j^2 - m_I^2)(s_{jk} + m_j^2)}{s_{IK}}$

Novel features in the Electroweak Sector

Resonance Matching

Branchings like $t \to bW,\ Z \to q\bar{q}$ etc.

- Large scales:
  EW shower offers best description
- Small scales:
  Breit-Wigner distribution

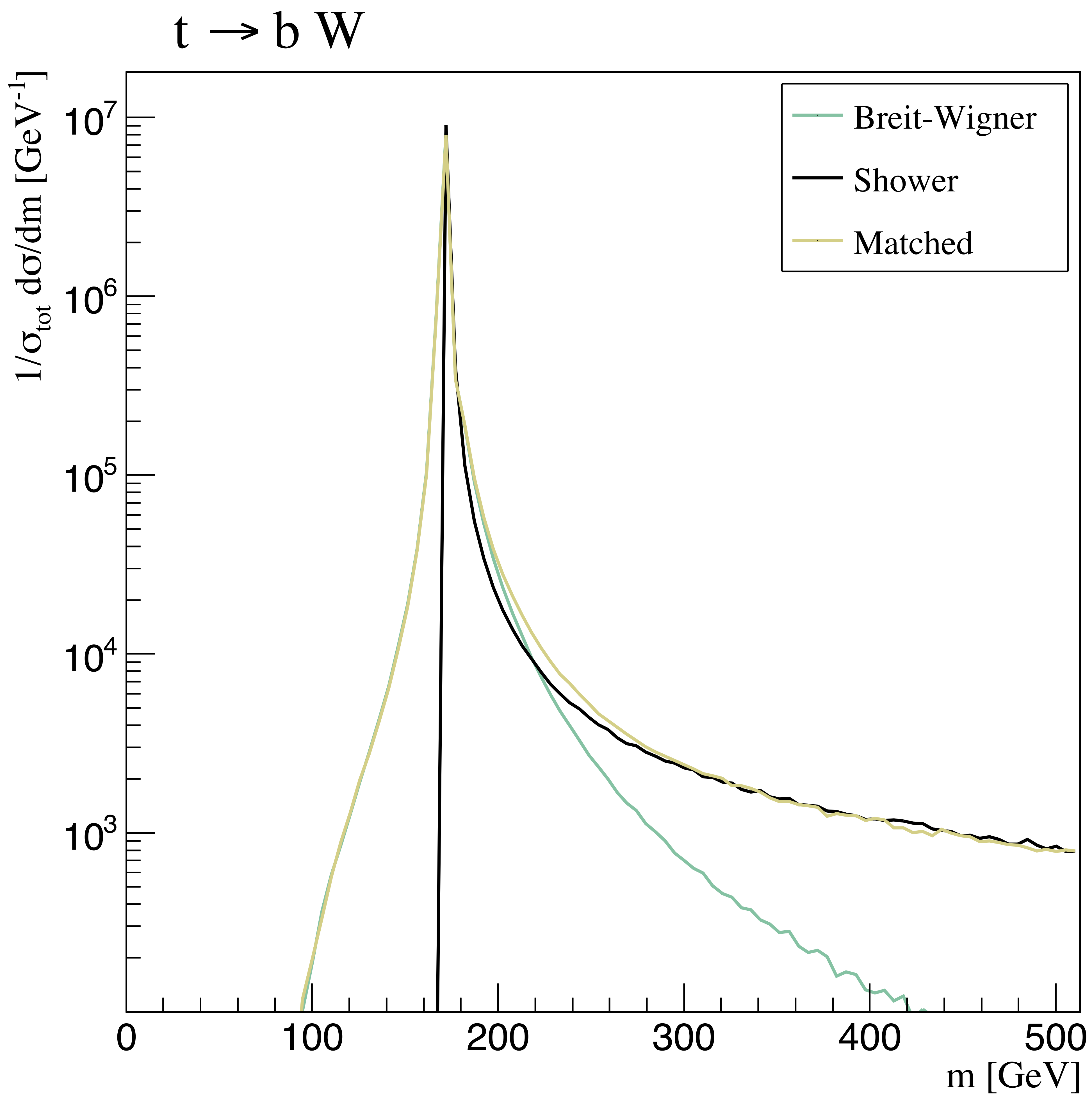
$$\mathrm{BW}(Q^2) \propto \frac{m_0\Gamma(m)}{Q^4 + m_0^2\Gamma(m)^2}$$

Matching:

- Sample mass from Breit-Wigner upon production
- Suppress shower by factor

$$\frac{Q^4}{(Q^2 + Q_{\mathrm{EW}}^2)^2}$$

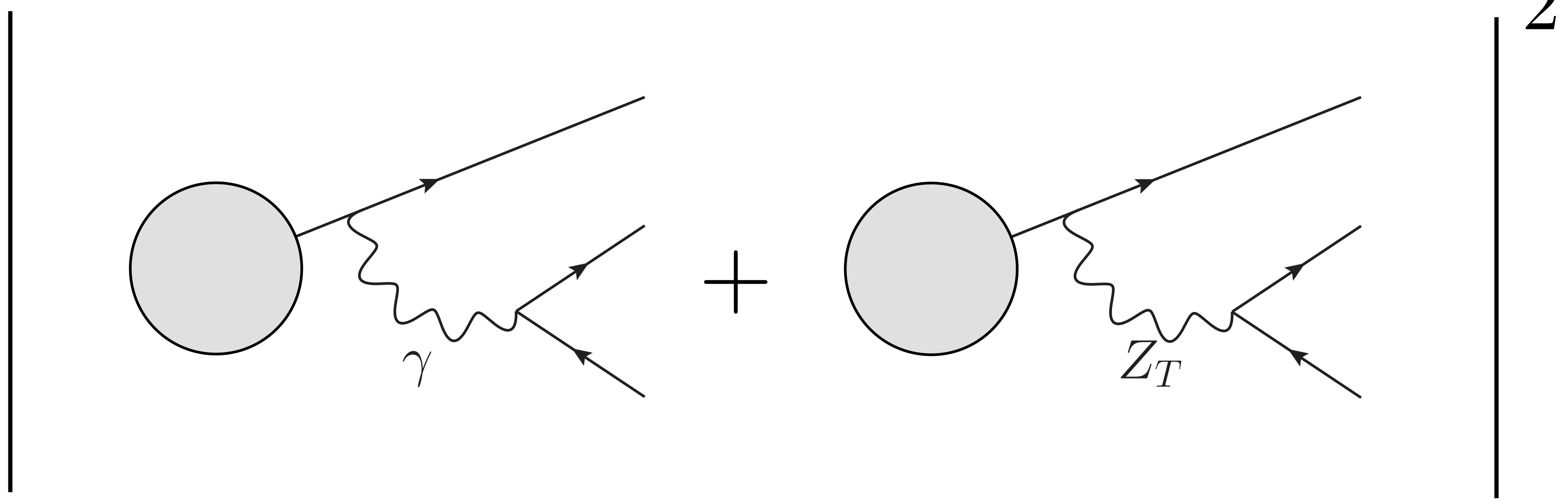
- Decay when shower hits off-shellness scale



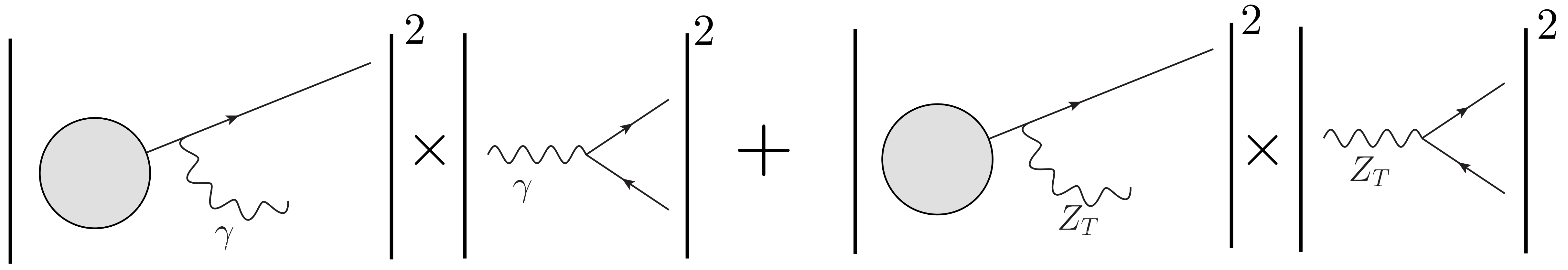
Neutral Boson Interference

Interference between γ, Z_T and h, Z_L

Physical contribution

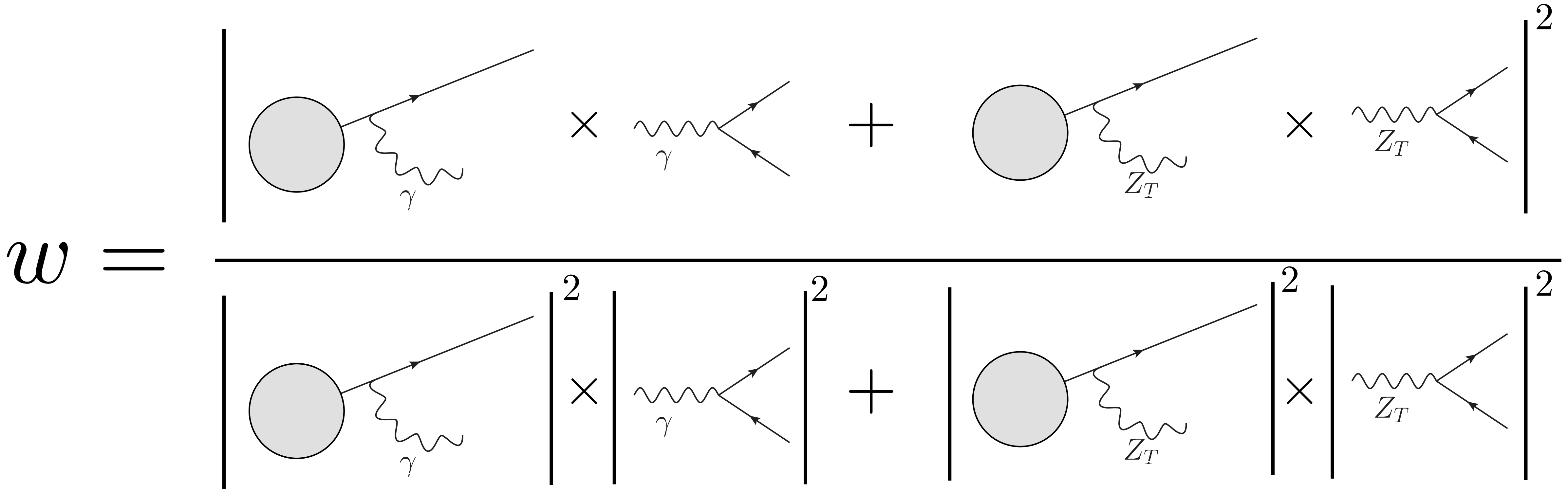


Shower approximation

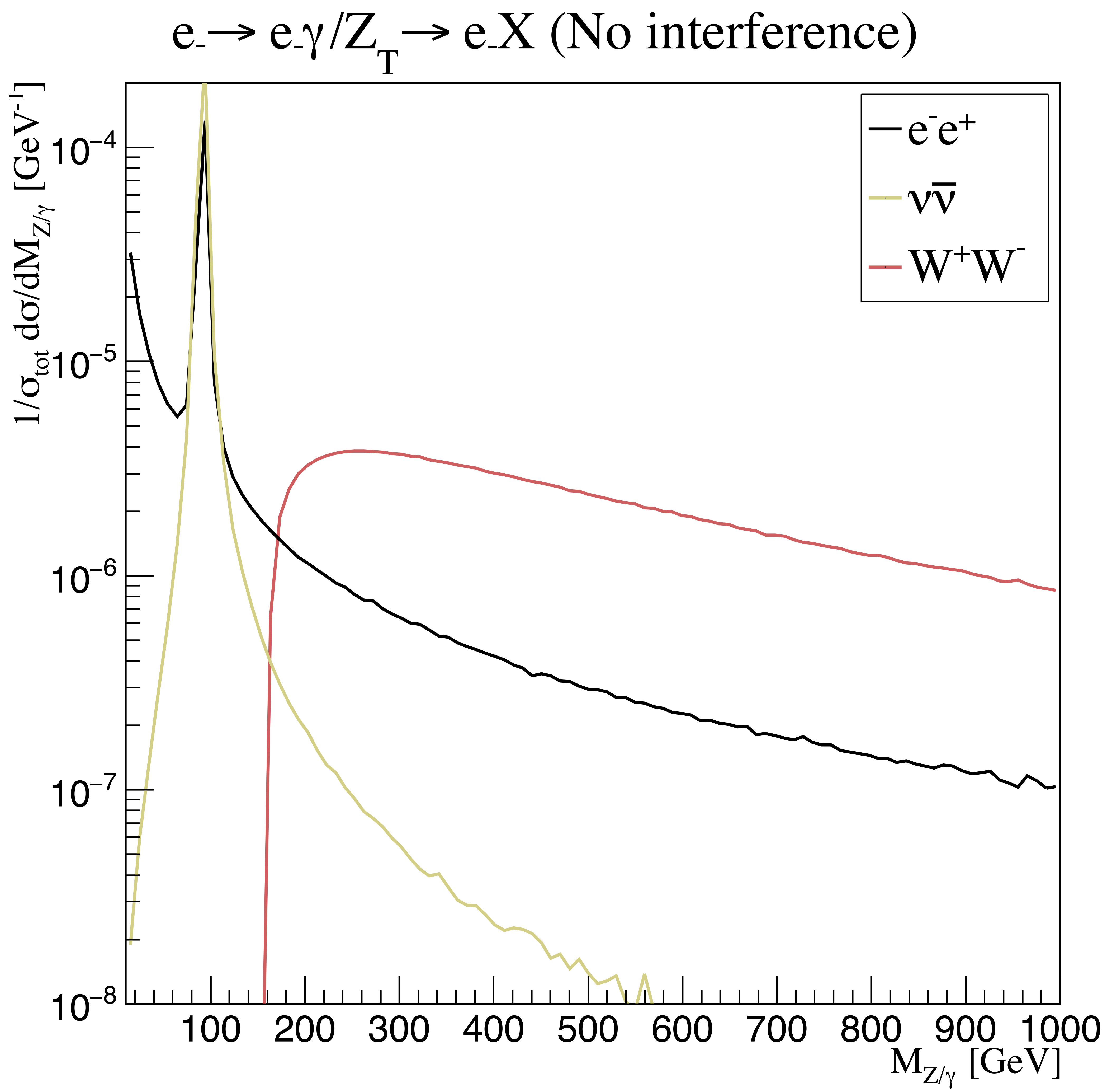


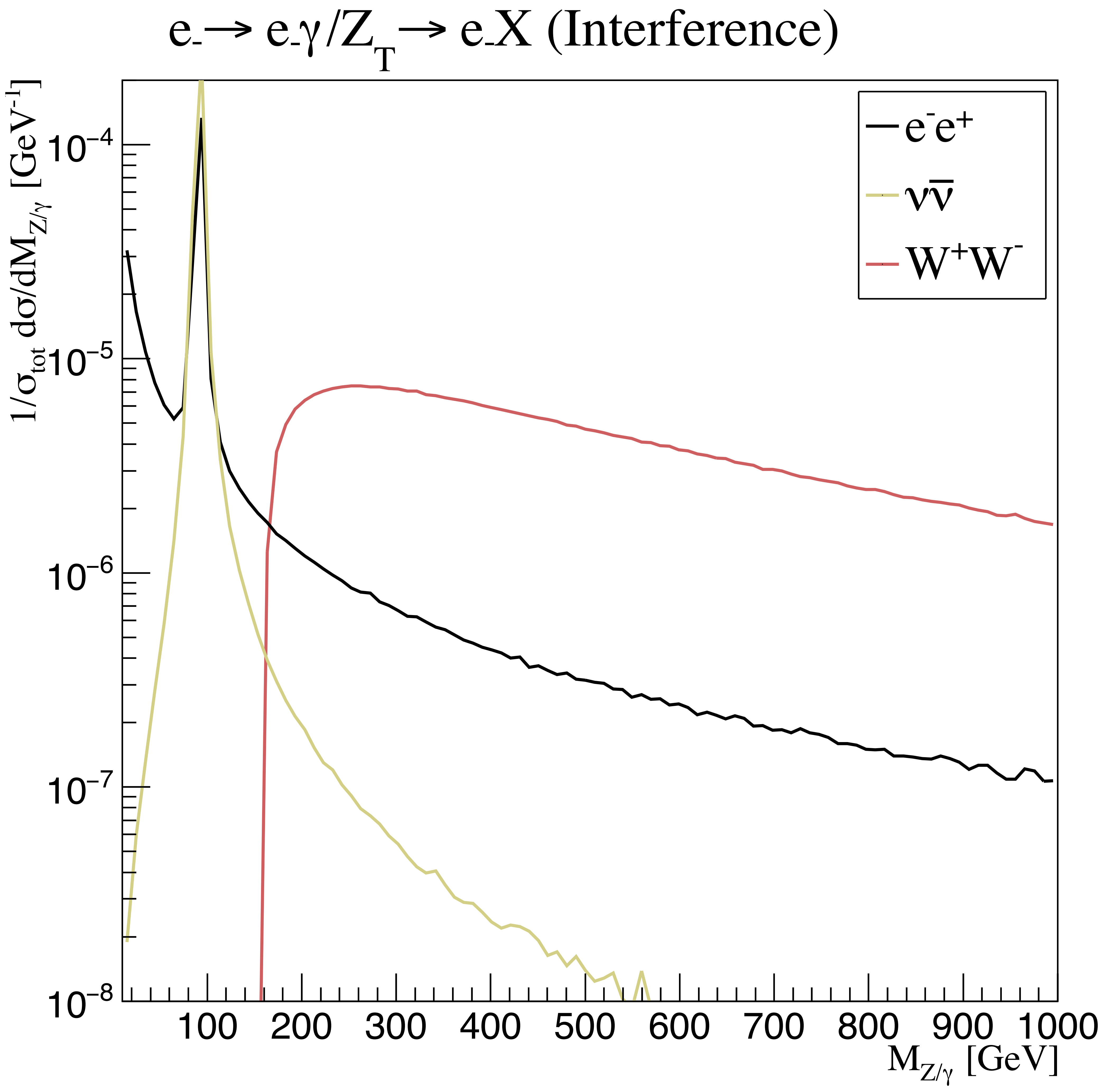
- Complicated solution: Evolve density matrices
  → Very computationally expensive

- Simple solution: Apply event weight
  → Does not get Sudakov right

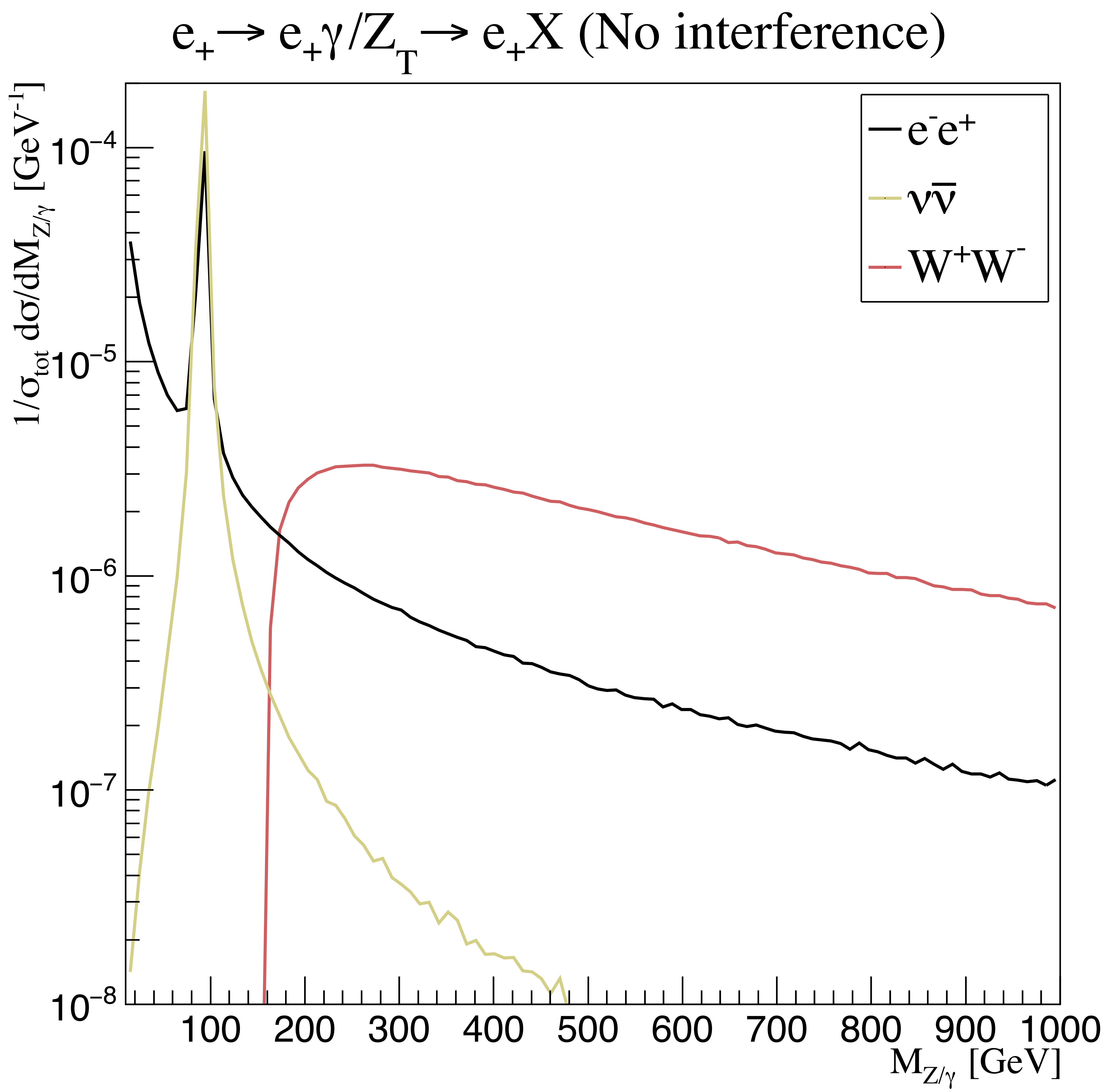


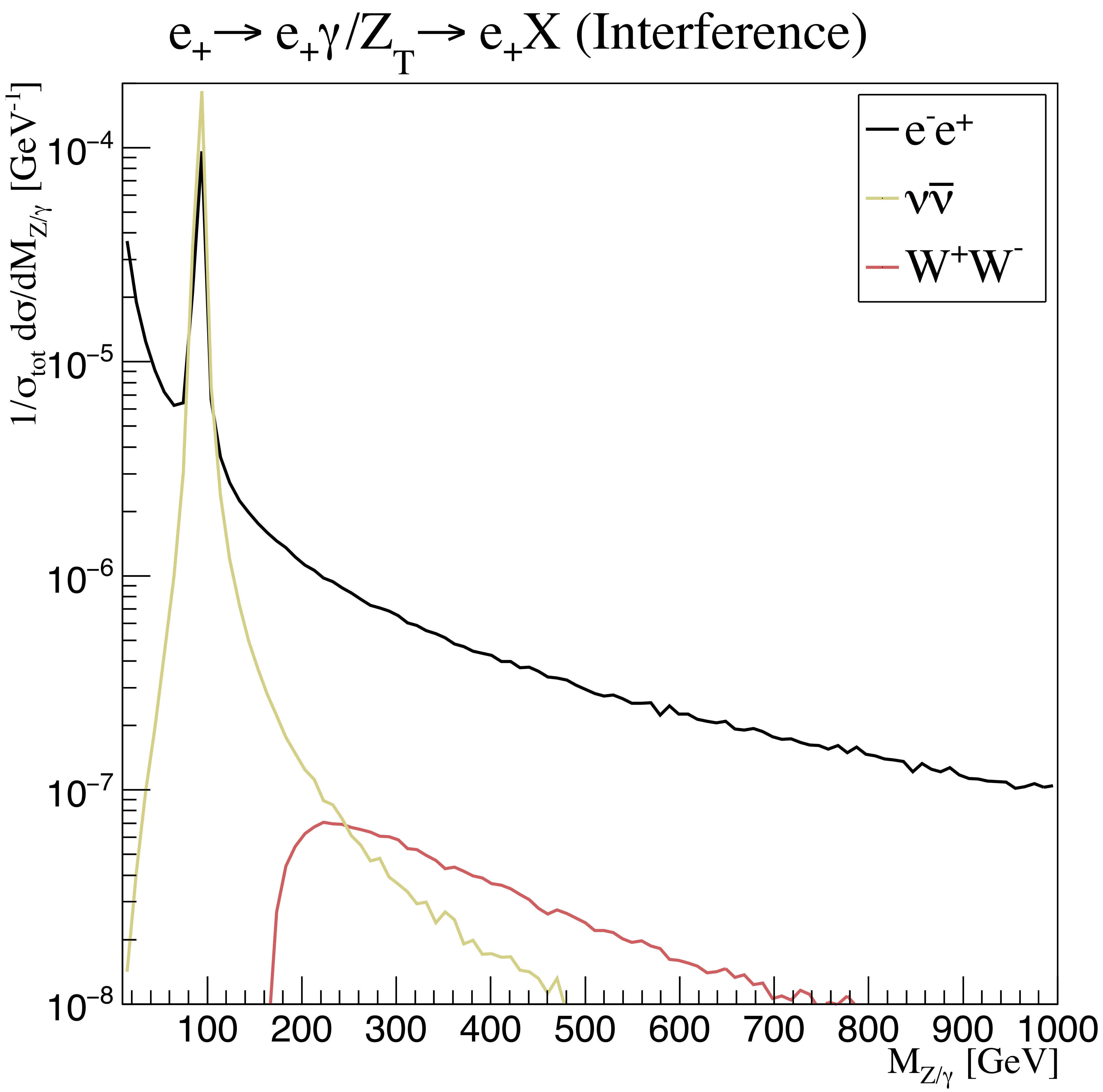
Bosonic Interference





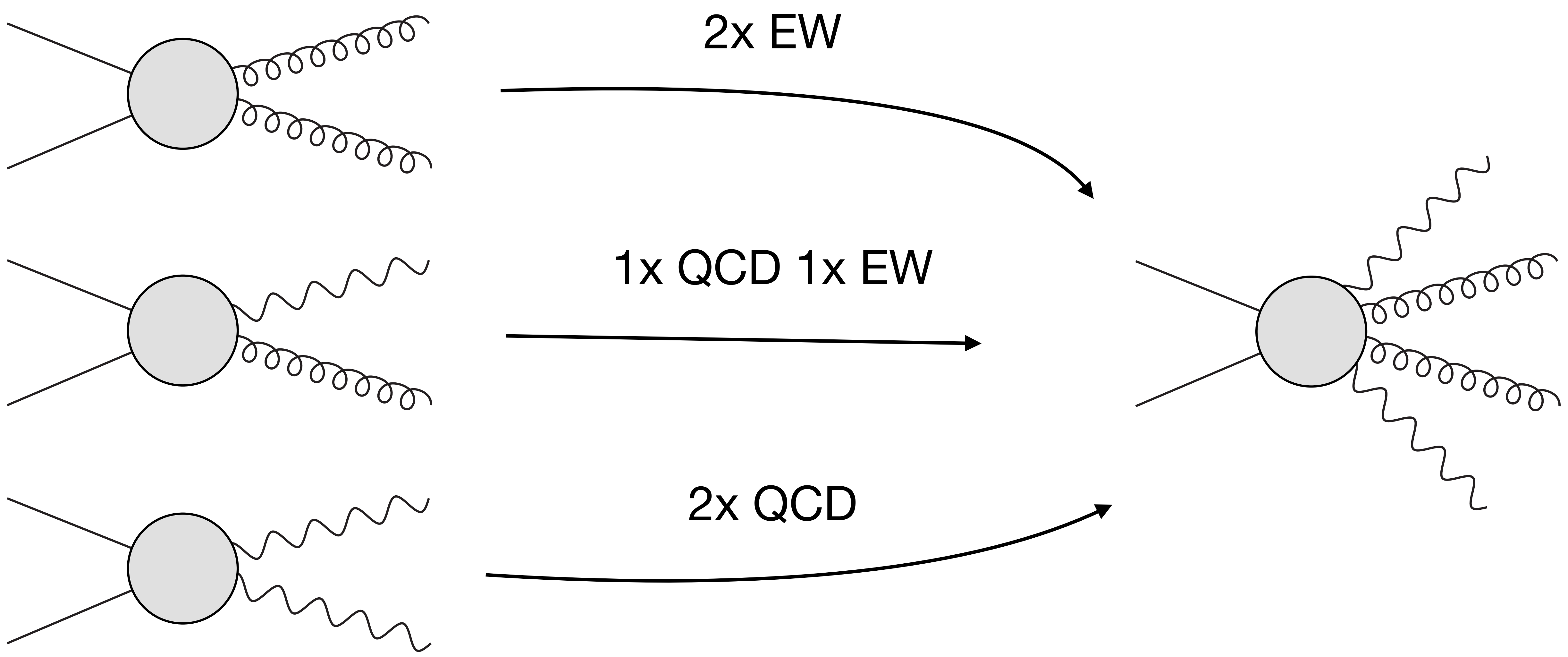
Bosonic Interference



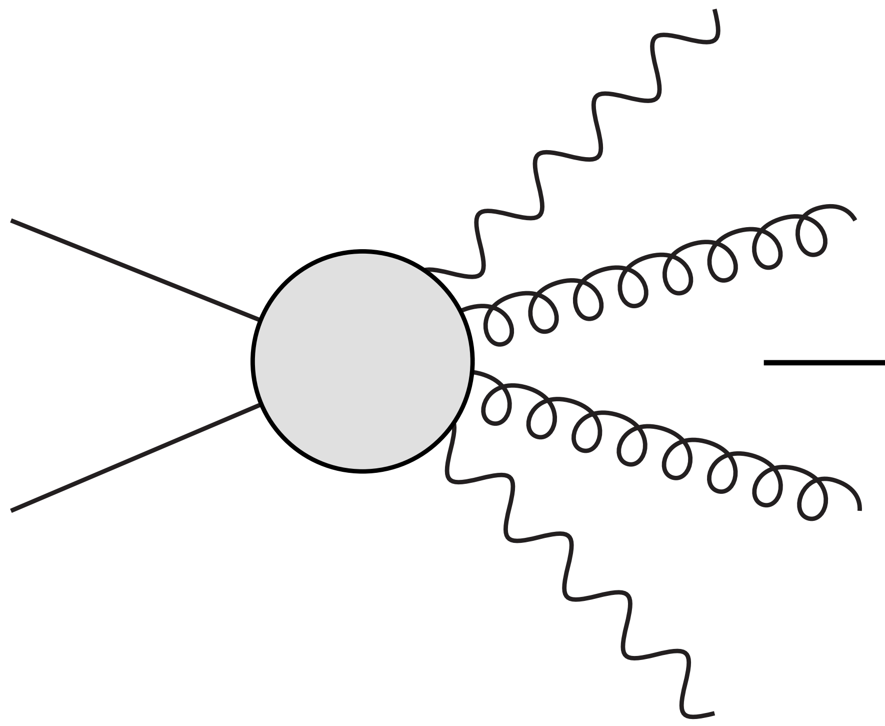


Overlap Veto

Double counting problem



Veto procedure

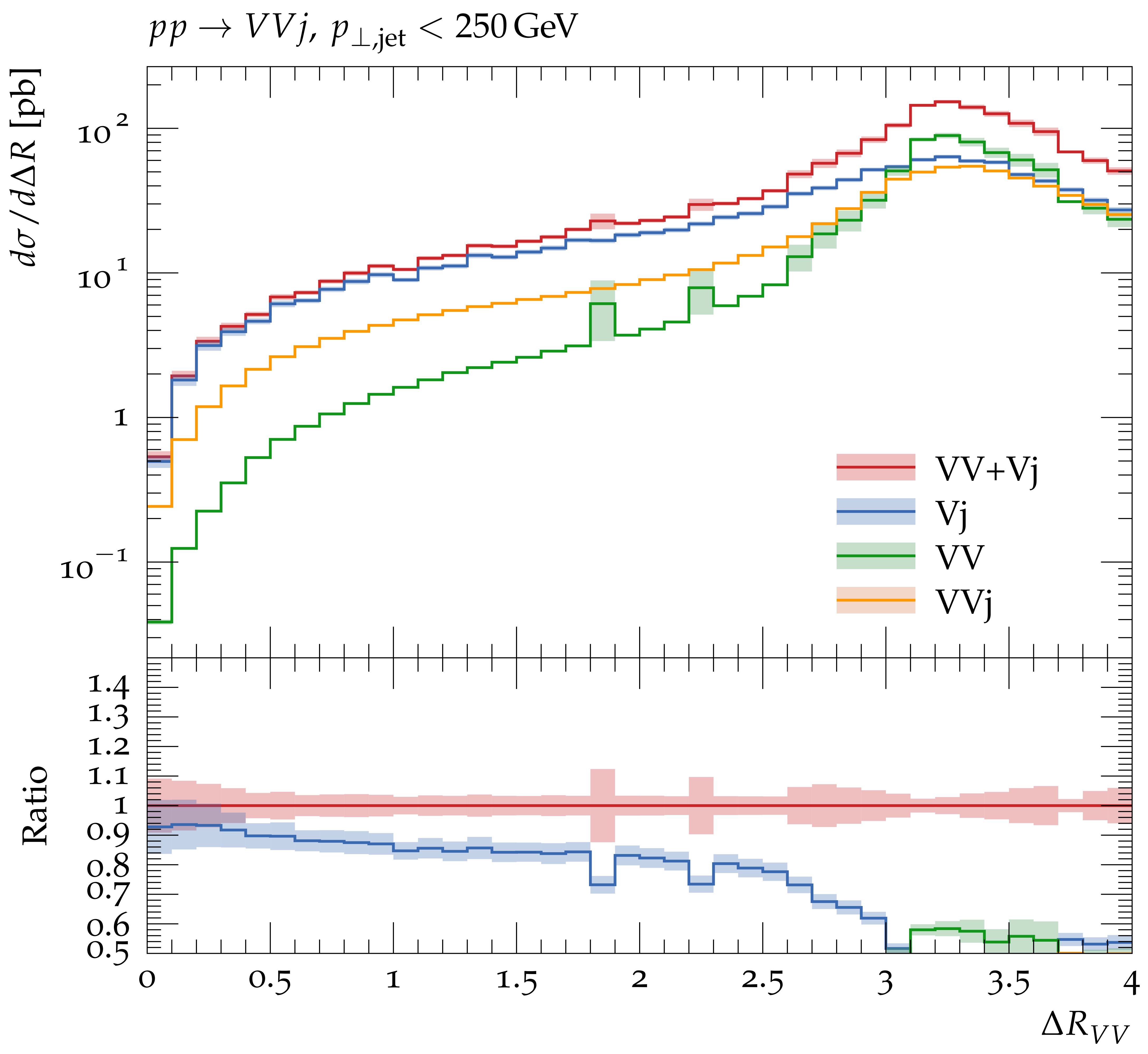


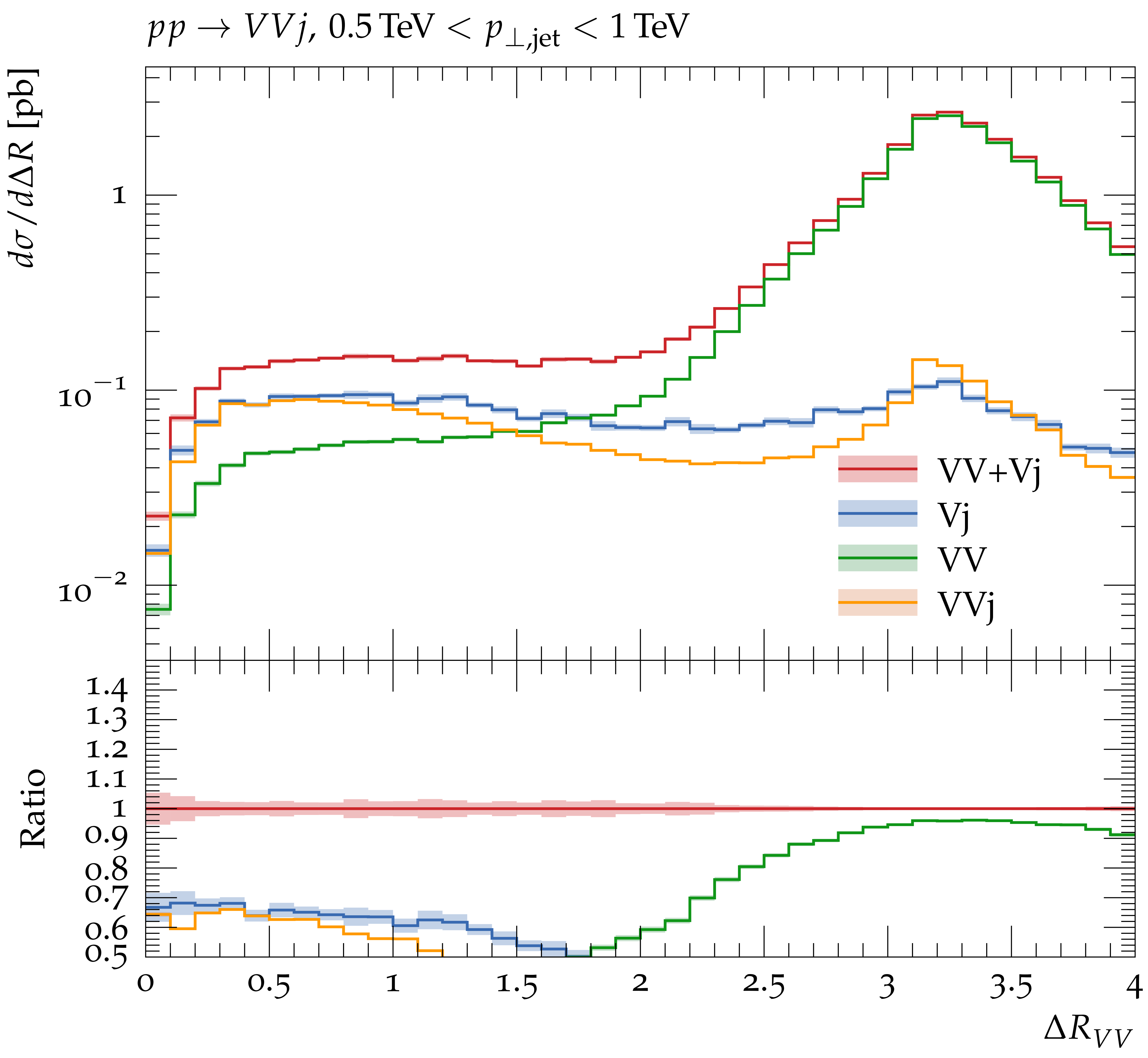
Last emission QCD? $\longrightarrow d_{ij}^{\text{Last}} < \min\left(d_{ij}^{\text{EW}}\right)$ → Accept Branching

Last emission EW? $\longrightarrow d_{ij}^{\text{Last}} < \min\left(d_{ij}^{\text{QCD}}\right)$ → Accept Branching

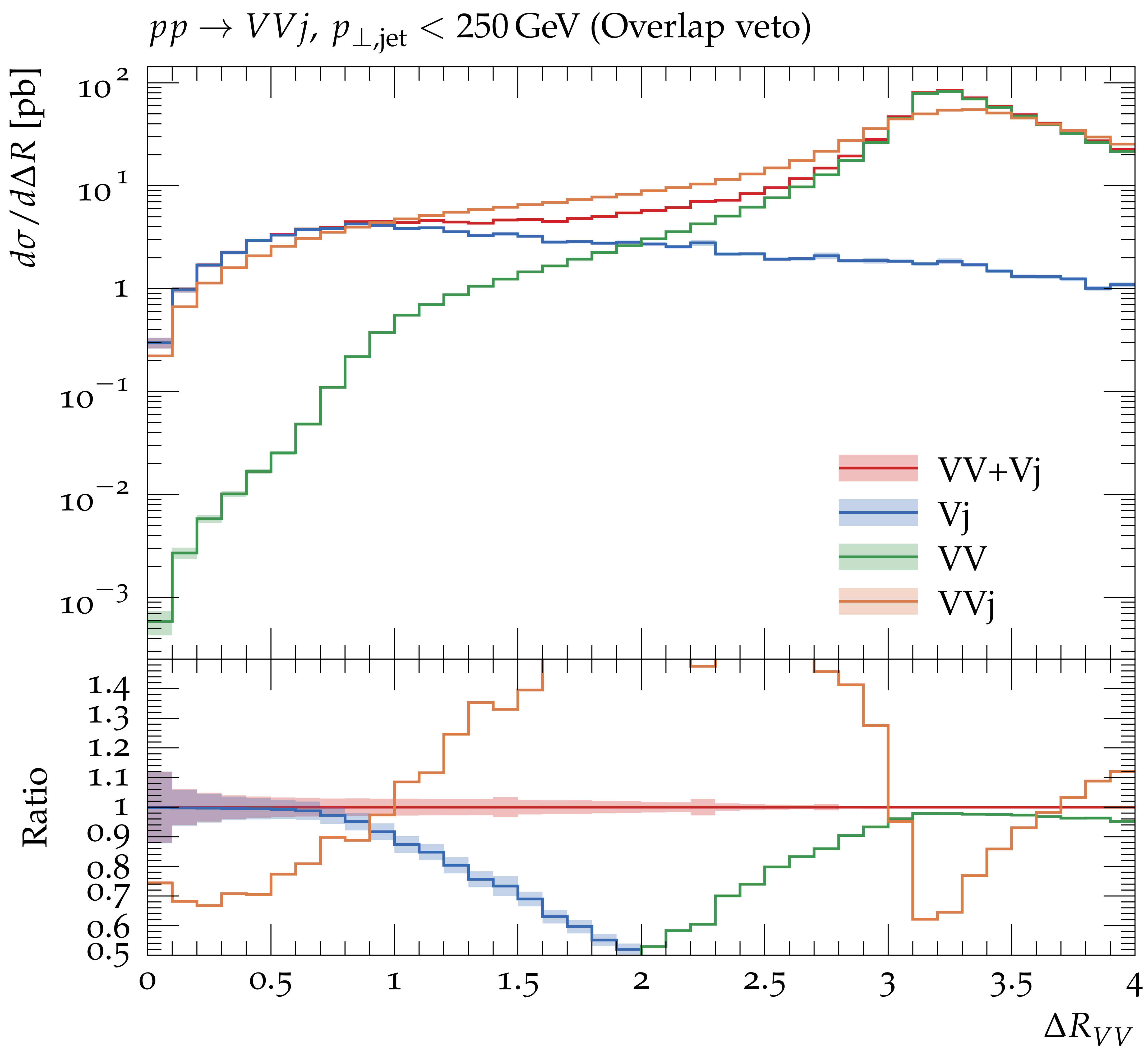
$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right)\frac{\Delta_{ij}}{R} + m_i^2 + m_j^2 - m^2$$

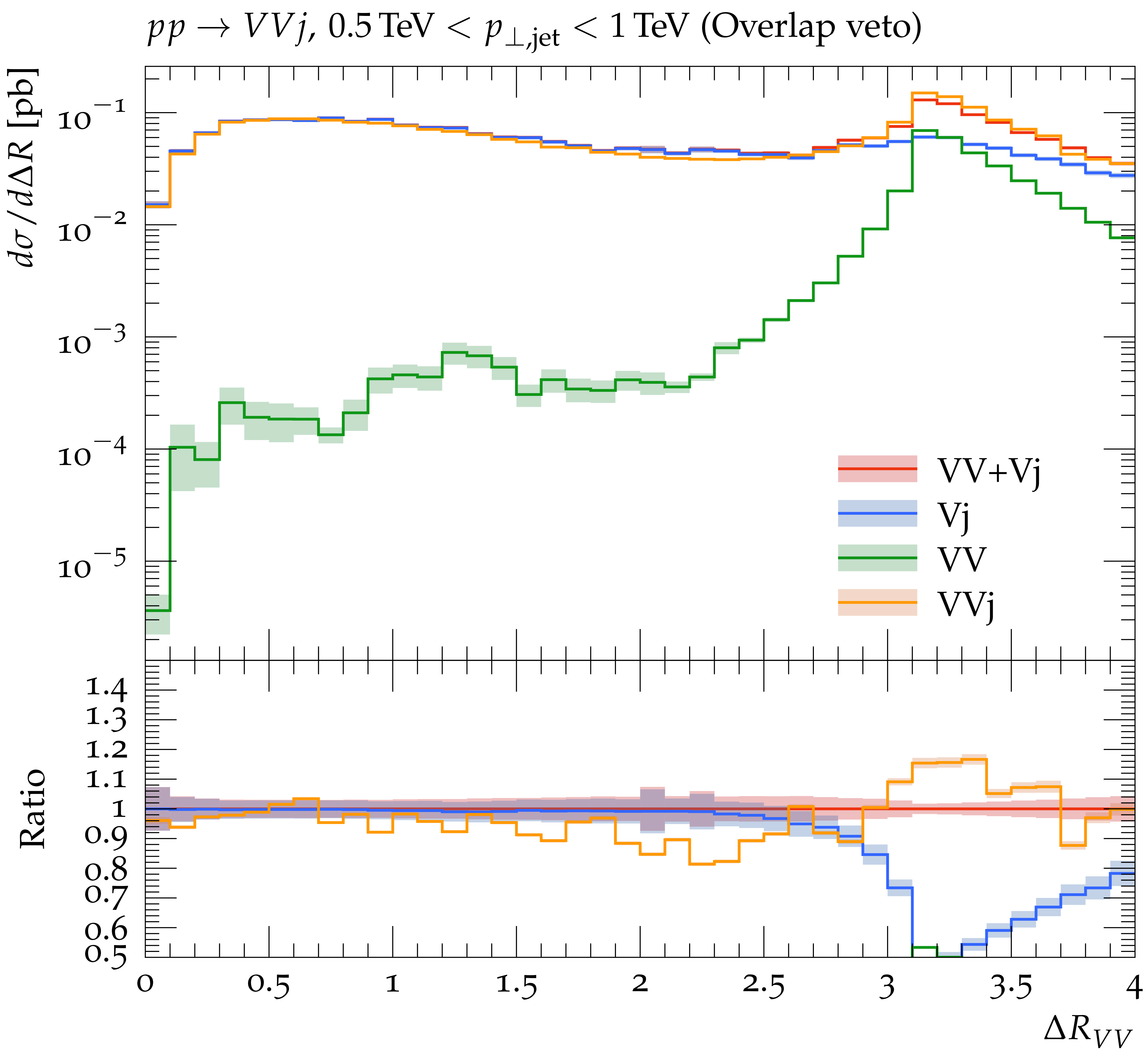
Overlap Veto





Overlap Veto



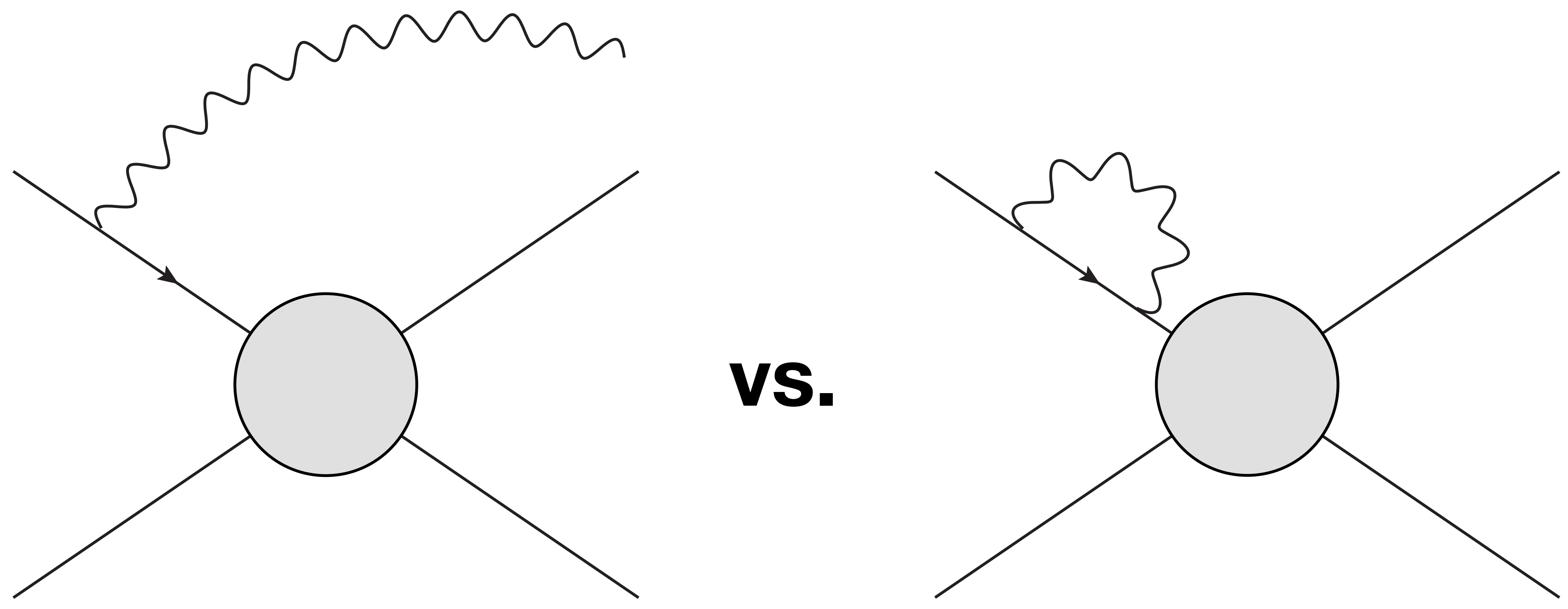


Bloch-Nordsieck Violations

BN / KLN Theorems: Real and virtual singularities cancel

Requirement: Summing over gauge indices

W radiation in the initial state:
PDFs are not isospin symmetric
→ Incomplete cancellation



Effects not large at LHC, but will be significant at higher energies

No straightforward solution in shower language

Conclusions

- Universal EW radiative corrections relevant at (HL)-LHC and future colliders
- EW sector offers rich physics, with lots of different collinear branchings
- Many features unique to the EW sector
  - Matching to resonance decays ✓
  - Neutral boson interference ✓
  - Overlap between hard scatterings ✓
  - Bloch-Nordsieck violations ✗
- EW shower will be publicly available as part of the Vincia shower
  Will be included in Pythia 8.3 out of the box

Backup

Spinor-Helicity formalism

Fermion

$$u_\pm(p) = \frac{1}{\sqrt{2p\cdot k}}(\not{p} + m)u_\mp(k)$$

$$v_\pm(p) = \frac{1}{\sqrt{2p\cdot k}}(\not{p} - m)u_\mp(k)$$

$k \longrightarrow$ helicity for massive fermions

Spin points in direction of motion

Gauge boson

$$\epsilon^\mu_\pm(p) = \pm\frac{1}{\sqrt{2}}\frac{1}{2p\cdot k}\bar{u}_\mp(k)\not{p}\gamma^\mu u_\pm(k)$$

$$\epsilon^\mu_0(p) = \frac{1}{m}\left(p^\mu - \frac{m^2}{p\cdot k}k^\mu\right)$$

$k \longrightarrow$ gauge choice

Purely transverse & longitudinal

$$k = (1, -\vec{e}_p)$$

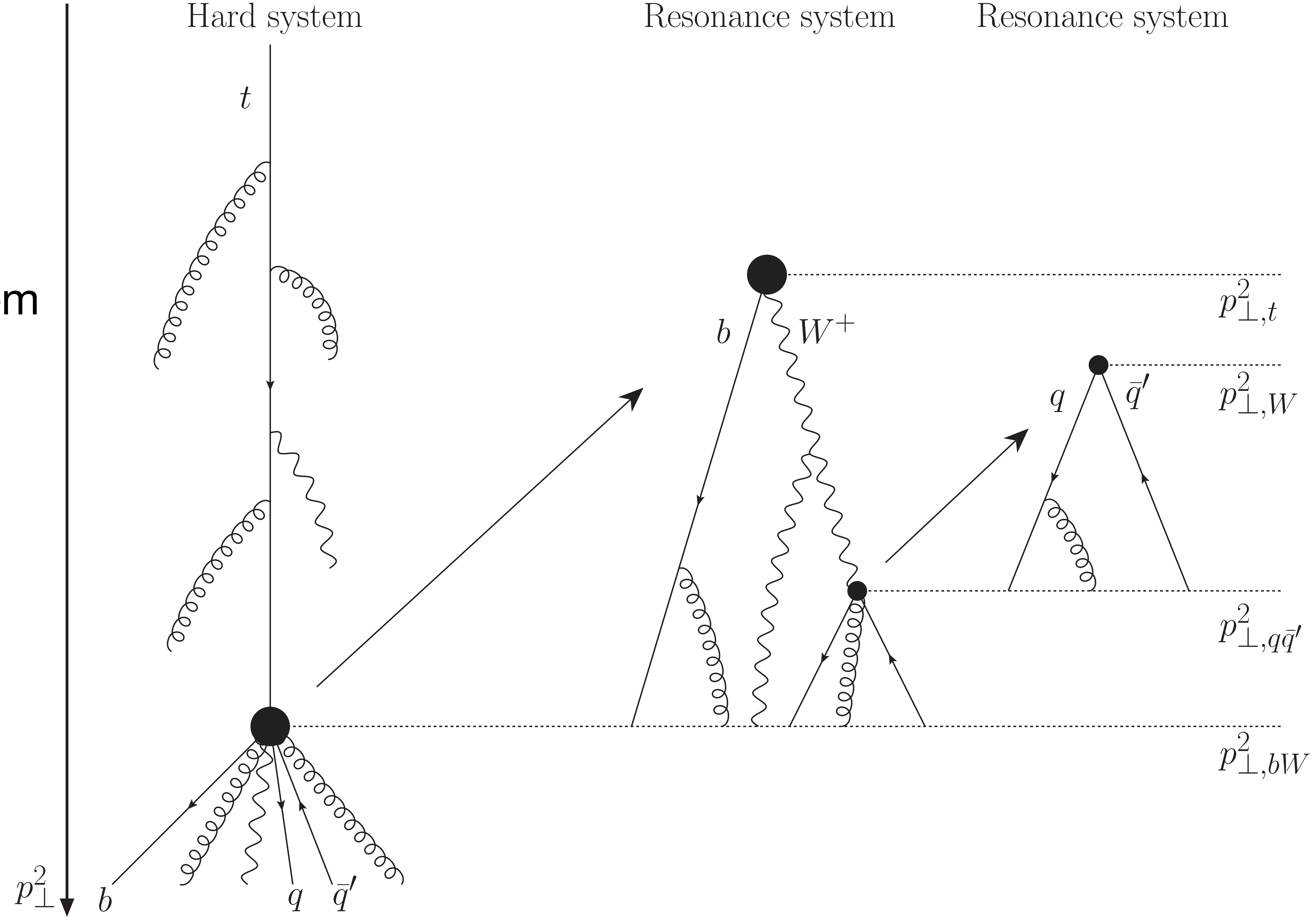
Resonance Matching

Pythia

- Narrow width approximation
- Decay showers after hard system

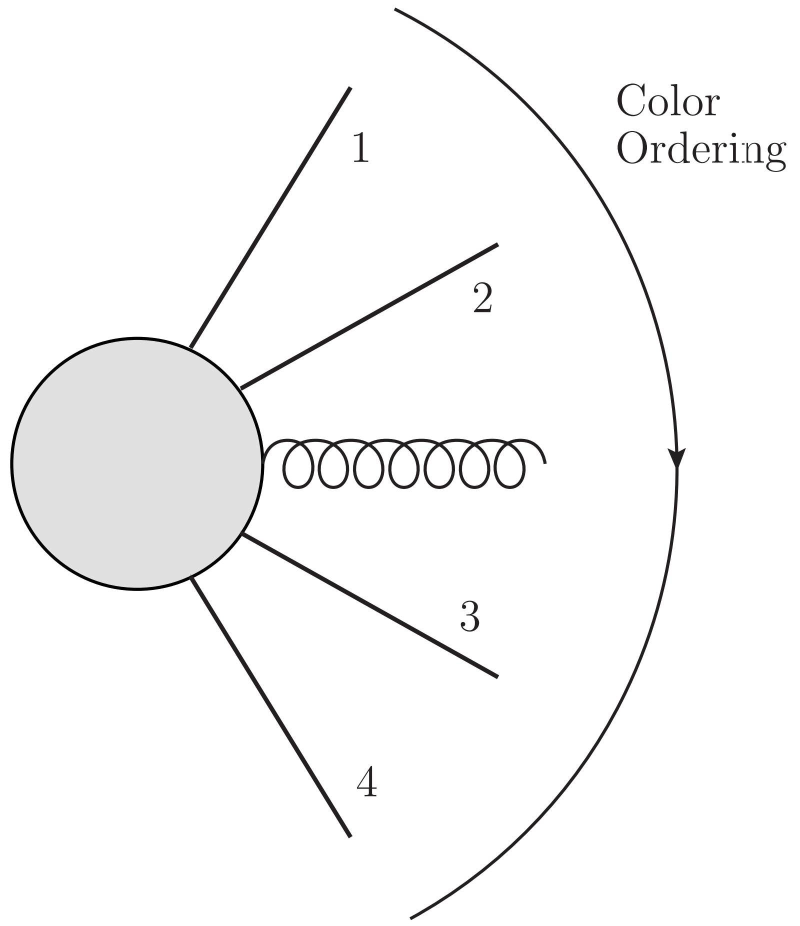
Vincia

- Decays part of hard system
- Natural treatment of finite width effects



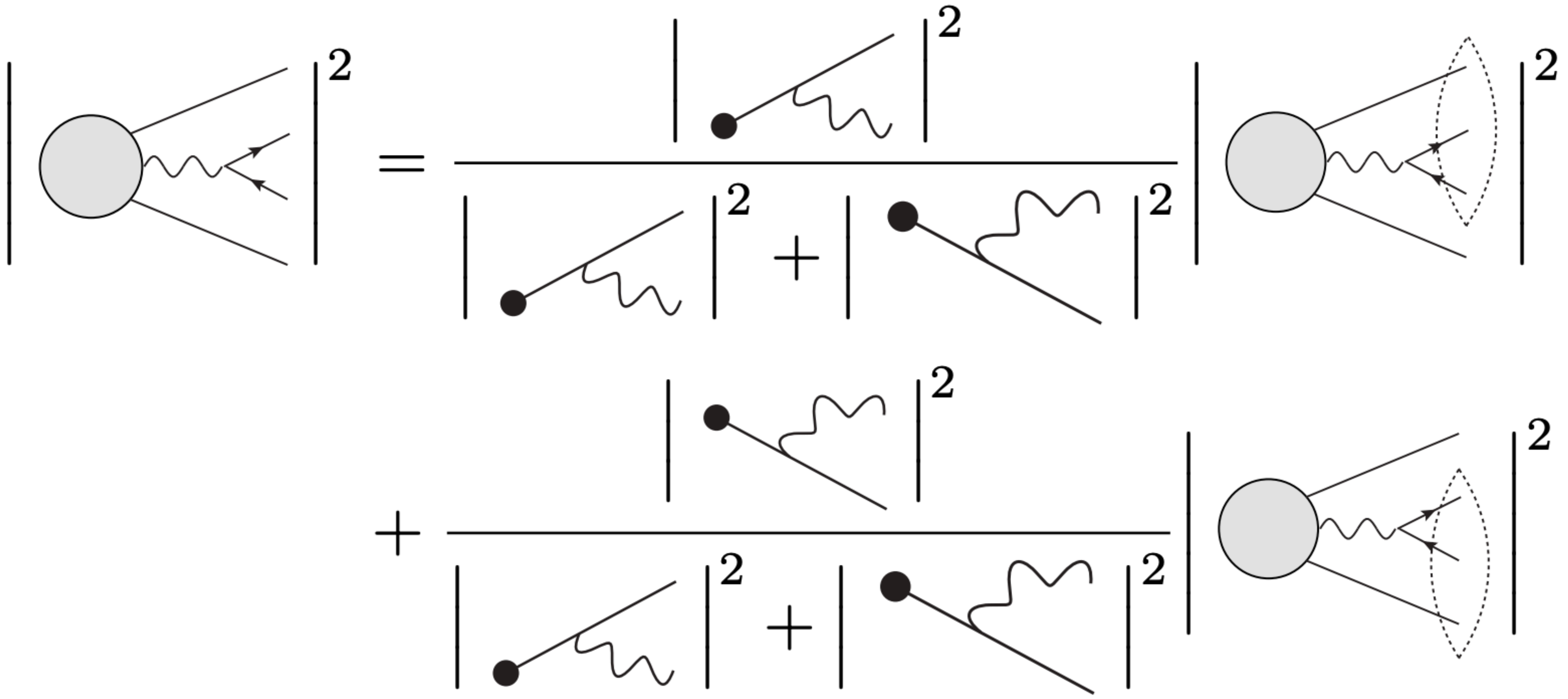
Recoiler Selection

In QCD recoiler determined
by colour structure



Gluon splitting: recoiler ambiguous

In EW no such guidance exists



Probabilistic choice to avoid
back reaction effects