Electroweak Corrections in the Vincia Parton Shower

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In collaboration with Ronald Kleiss, Peter Skands, Helen Brooks

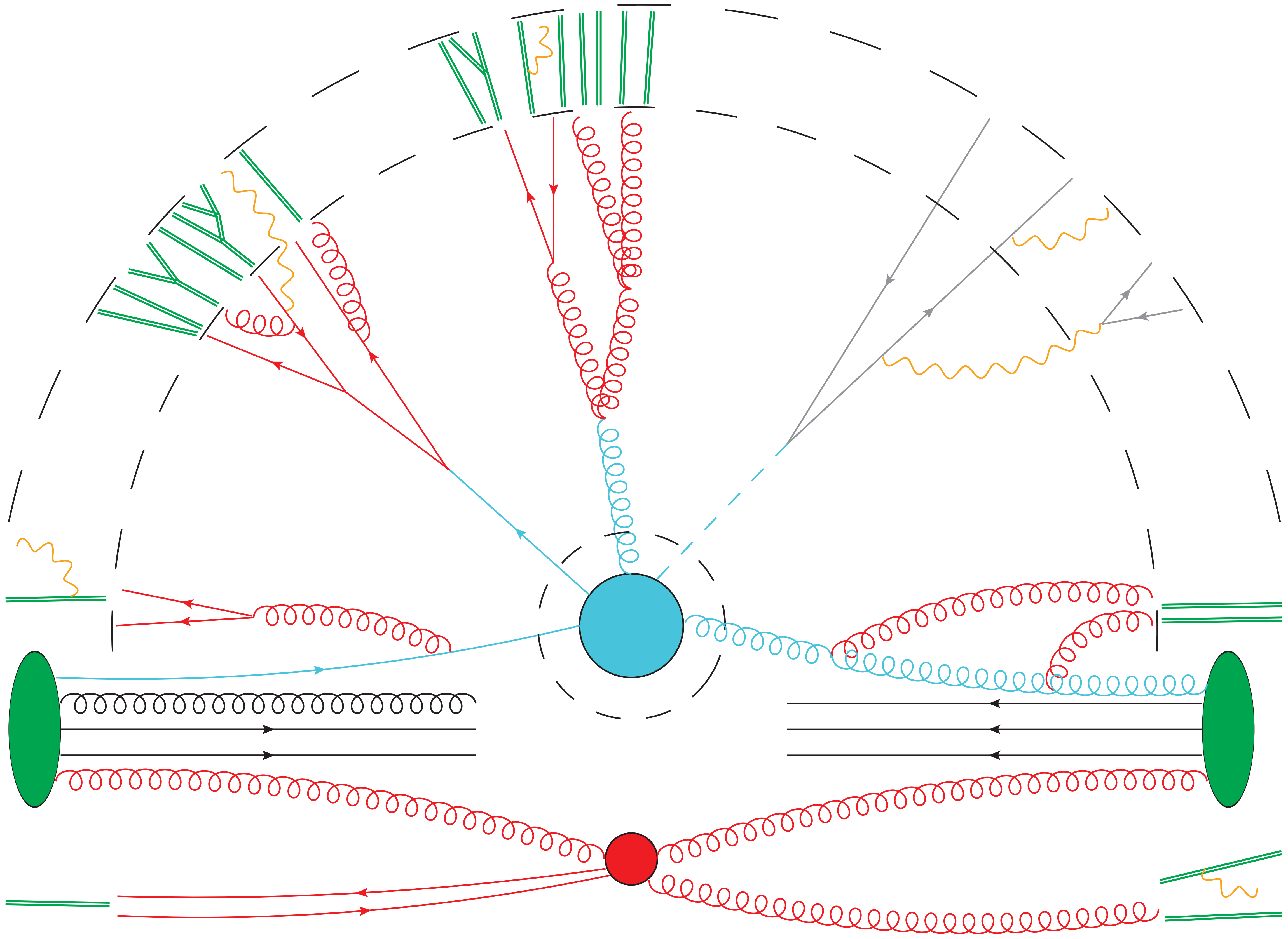
Overview

1. Parton shower overview
2. Electroweak showering
3. Novel features in the electroweak sector

Parton shower overview

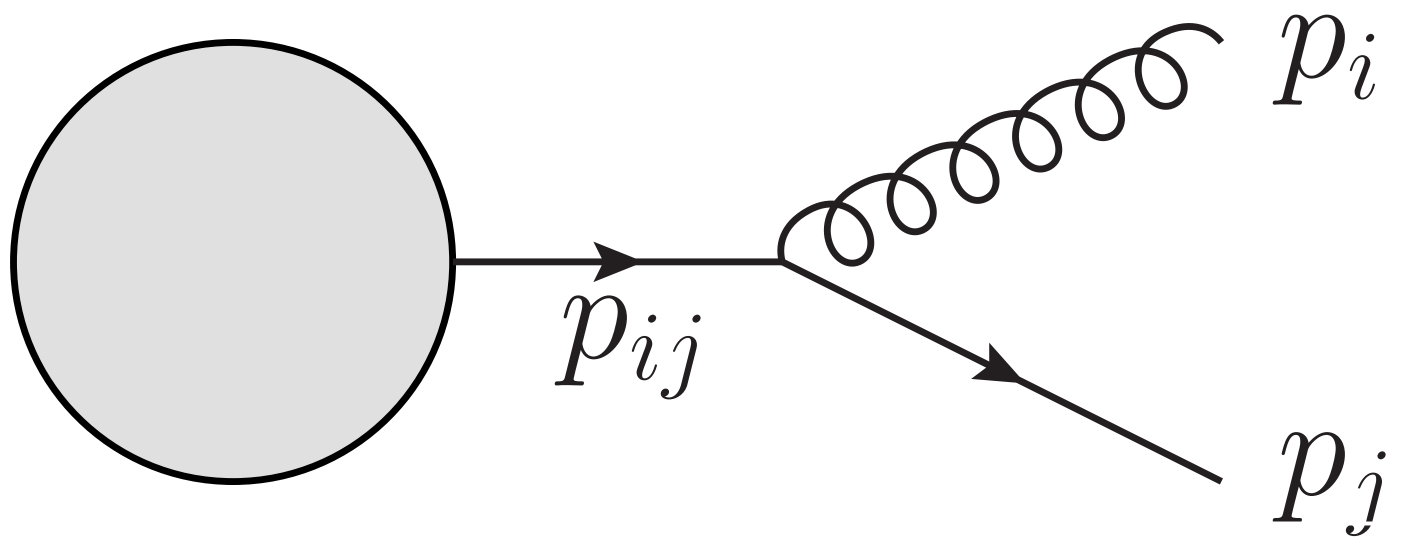
Parton Showers

- Essential part of Monte Carlo event generators
- Process-independent resummation framework
- Fully differential
- Interface hard scattering (high scale) to hadronization (low scale)
- Many types with many differences



Factorization

Based on factorisation properties of Matrix Element
in singular limits



1. Quasi-collinear limit

$p_i{\cdot}p_j \approx m_i^2, m_j^2$ and $E_i^2, E_j^2 \gg p_i{\cdot}p_j$

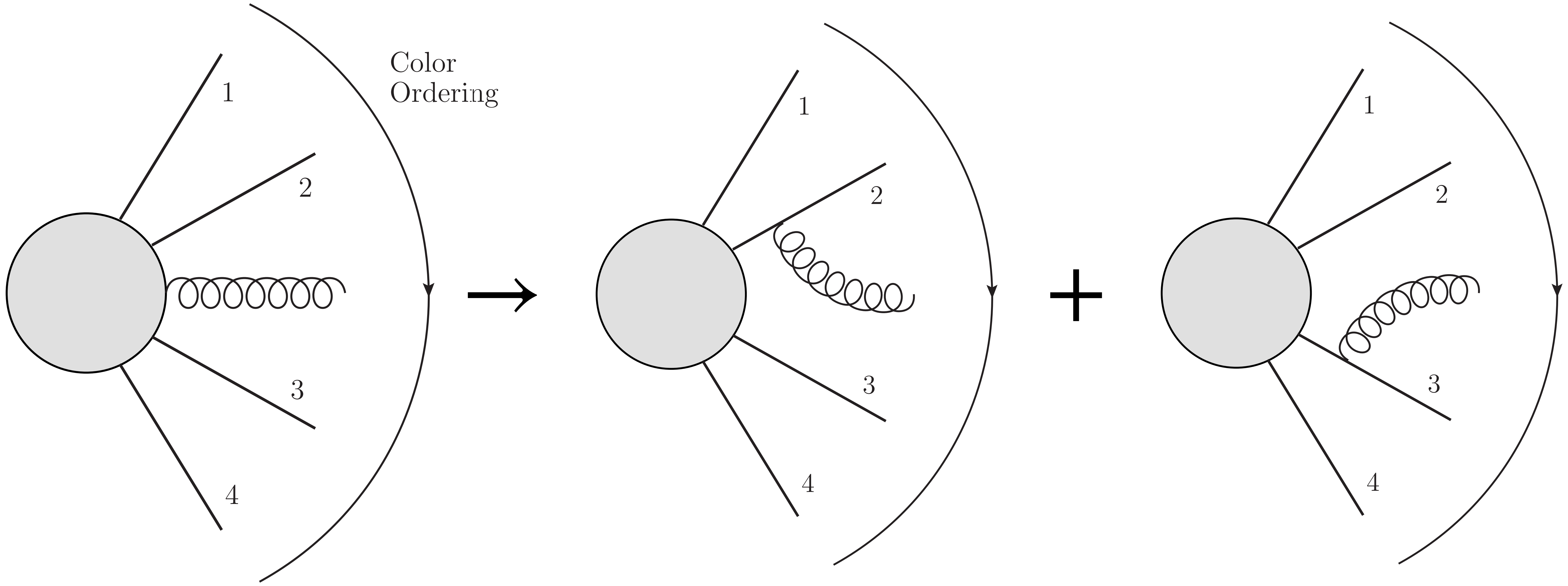
$$|M_{n+1}(..,p_i,p_j,..)|^2 \to 8\pi\alpha_s \frac{1}{(p_i+p_j)^2} P_{(ij)\to ij}(z)|M_n(..,p_{ij},..)|^2$$

Factorization

Based on factorisation properties of Matrix Element in singular limits

2. Soft limit

$E_j \approx m_j$ and $E_i, E_k \gg E_j$



$$|M_{n+1}(.., p_i, p_j, p_k..)|^2 \to 4\pi\alpha_s C\left[2\frac{p_i \cdot p_k}{p_i \cdot p_j p_j \cdot p_k} - \frac{m_i^2}{(p_i \cdot p_j)^2} - \frac{m_k^2}{(p_j \cdot p_k)^2}\right]|M_n(.., p_i, p_k..)|^2$$

$$+\mathcal{O}(1/N_C^2)$$

Main Ingredients

1. Phase space factorisation

$$d\Phi_{n+1} = d\Phi_n \times d\Phi_{\text{ps}}$$

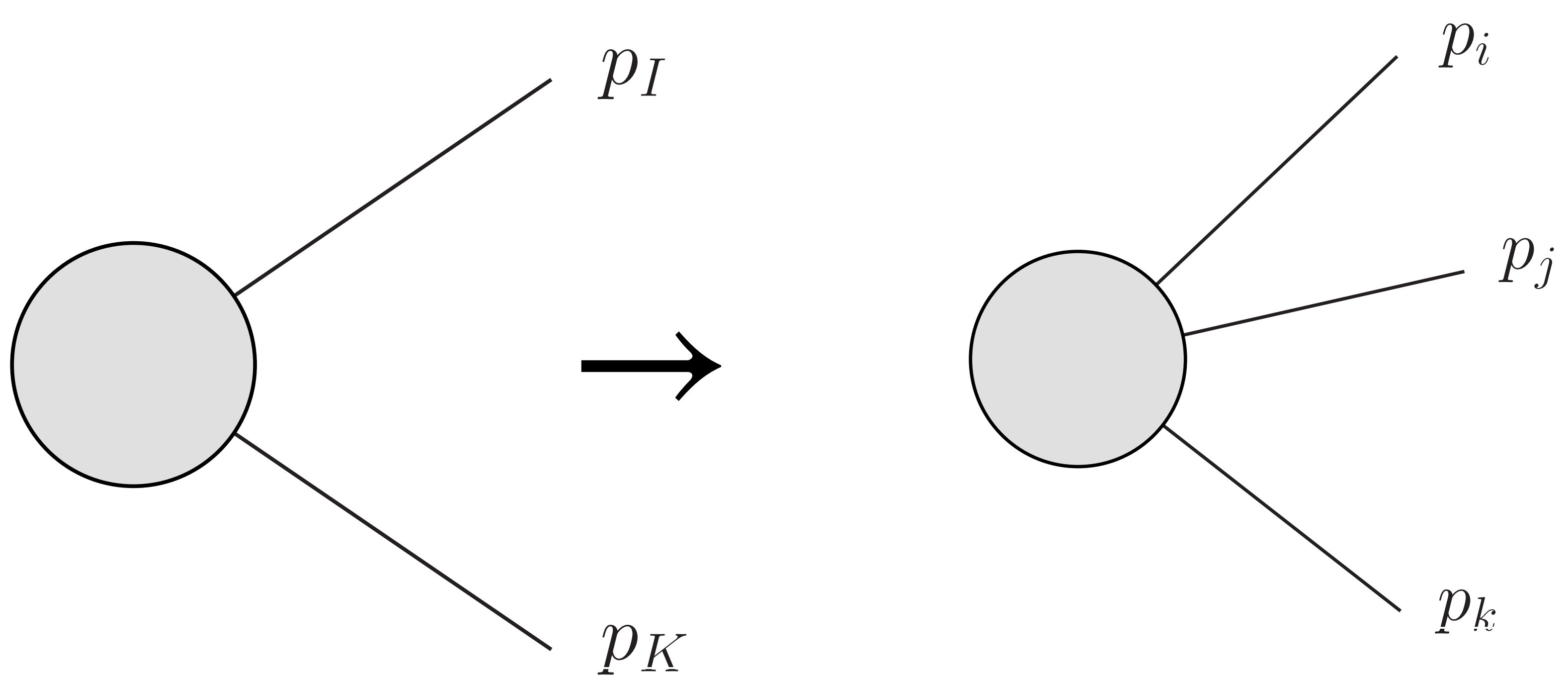
2. Ordering scale

$$p_\perp^2(\Phi_{\text{ps}})$$

3. Branching kernel

$$|M_{n+1}(\Phi_{n+1})|^2 \approx \sum_i B_i(\Phi_{\text{ps}}) \times |M_n(\Phi_n)|^2$$

→ Comes with a **kinematic map**



1. Momentum conservation
2. IR safety

Parton Showers

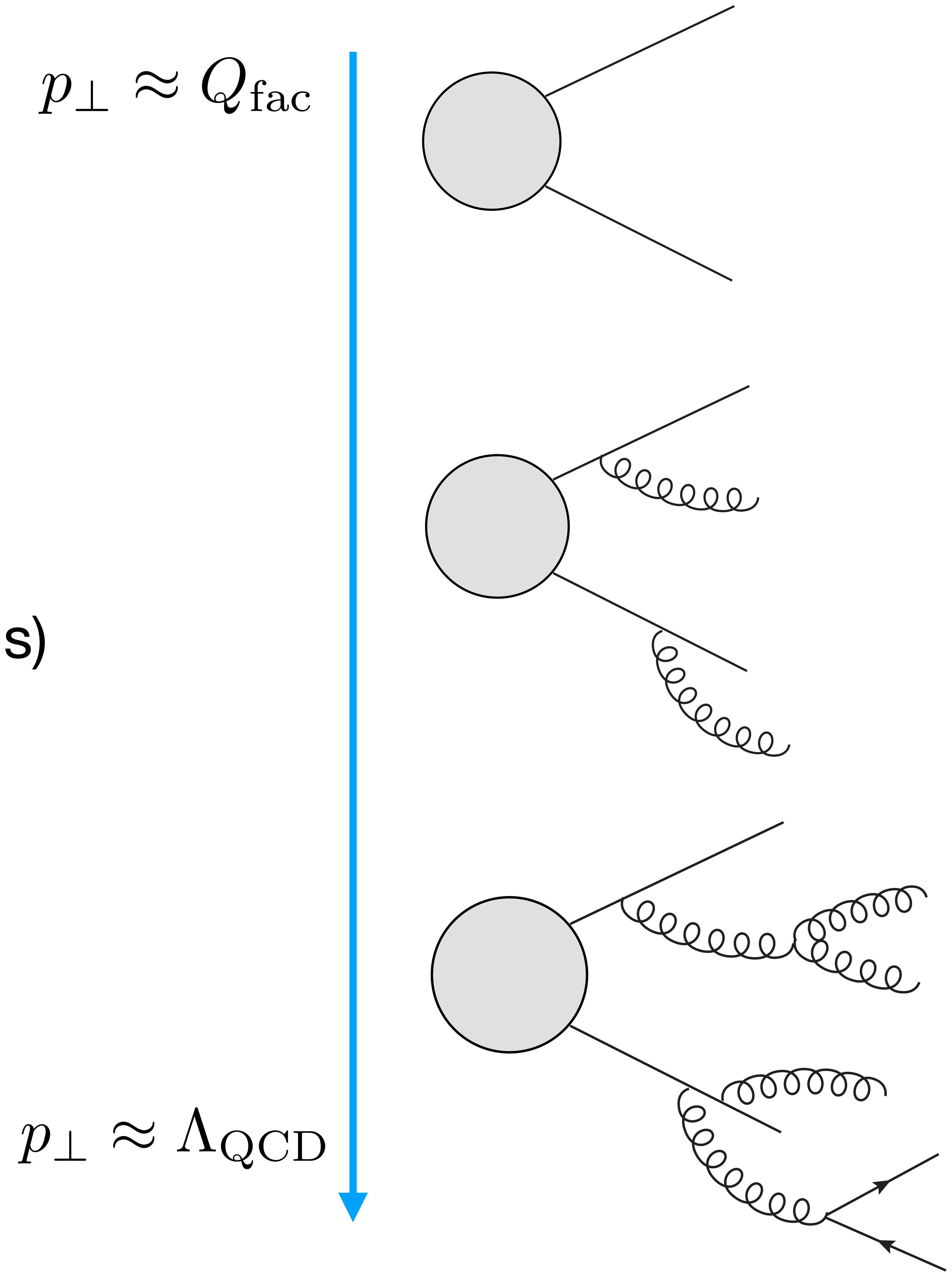
Branching kernel (real corrections)

$$\boxed{P_i(\Phi_{\text{ps},i}) = B(\Phi_{\text{ps},i})\Theta(p^2_{\perp,i} < p^2_{\perp,i-1}) \times \Delta(p^2_{\perp,i-1}, p^2_{\perp,i})}$$

Sudakov factor (virtual corrections)

$$\Delta(p^2_{\perp,i-1}, p^2_{\perp,i}) = \exp\left(-\int_{p^2_{\perp,i}}^{p^2_{\perp,i-1}} d\Phi_{\text{ps}} B(\Phi_{\text{ps}})\right)$$

Parton shower is *unitary*:
cancellation of real and virtual corrections
$\longrightarrow \sigma_{\text{inc}}$ unaltered



Parton Shower Accuracy

- Formal NLL accuracy

  Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez 2002.11114
  Nagy, Soper 2011.04773
  Forshaw, Holguin, Platzer 2003.06400

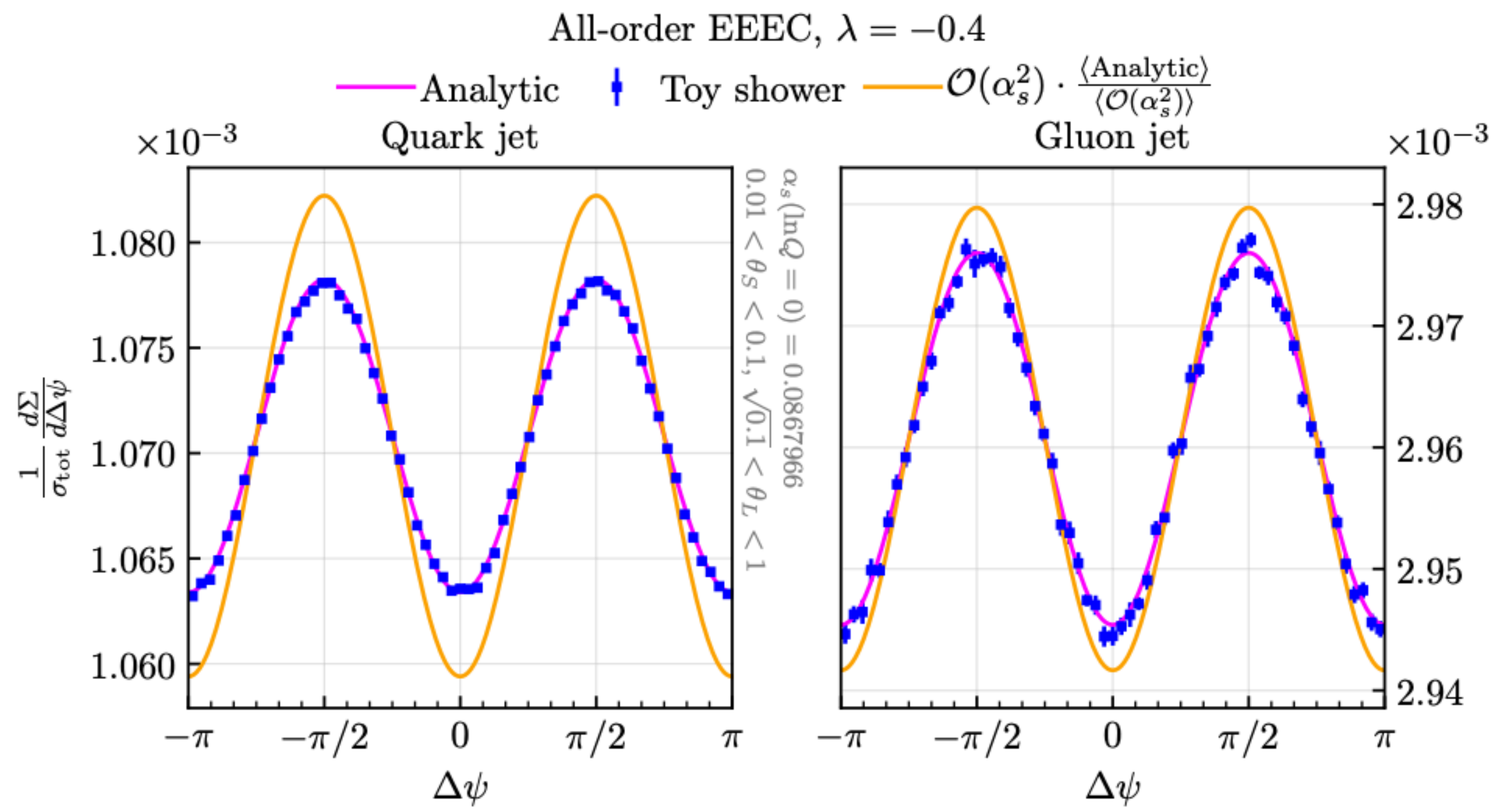
- Inclusion of higher-order branching kernels

  → Requirement for NNLL

  Hoche, Krauss, Prestel 1705.00982
  Li, Skands 1611.00013

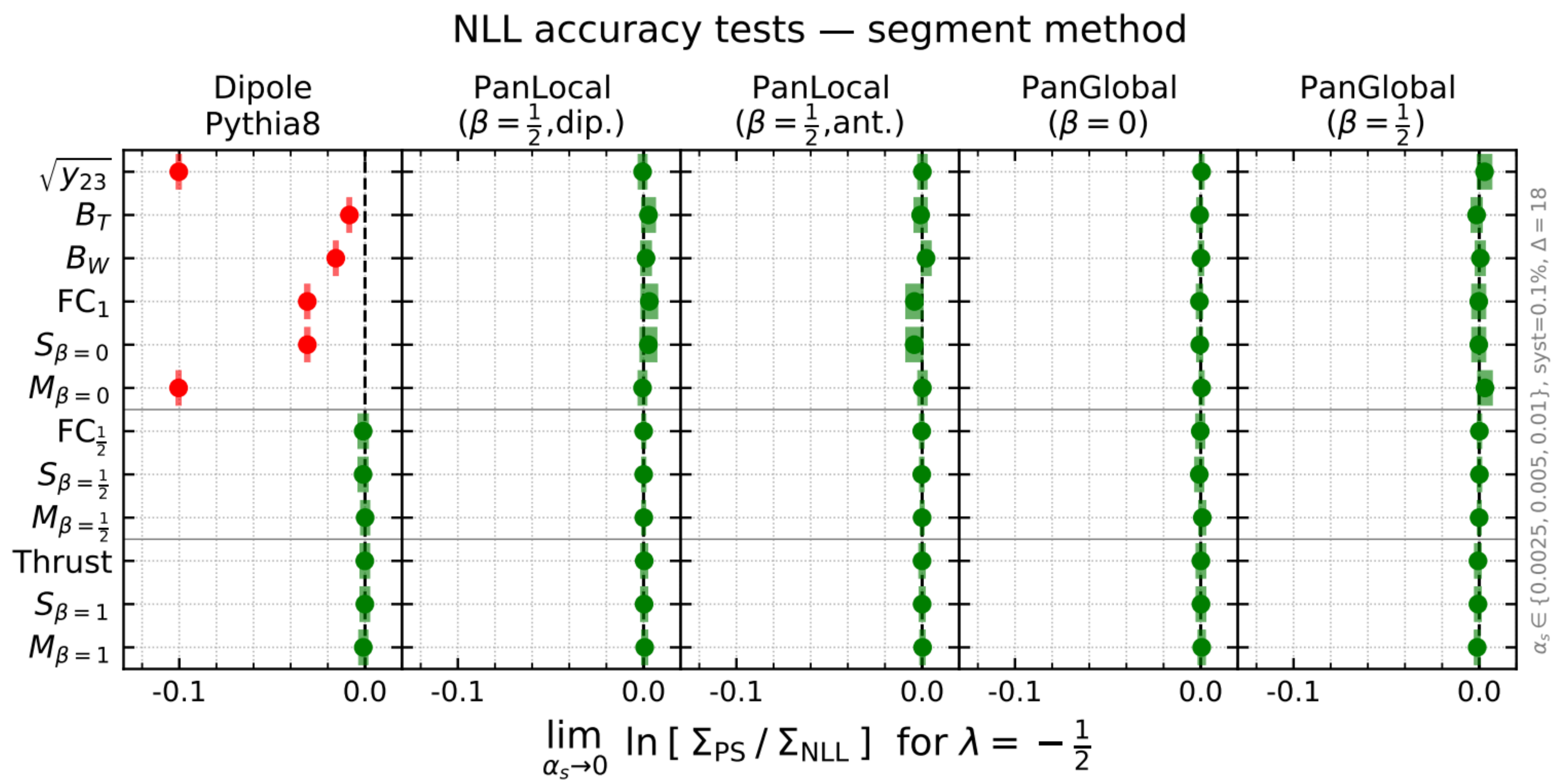
- Spin correlations

  Karlberg, Salam, Scyboz, RV 1611.00013
  Richardson, Webster 1807.01955



- Subleading colour effects $1/N_c^2 \sim 10\%$

  Hamilton, Medves, Salam, Scyboz, Soyez 2011.10054
  Nagy, Soper 1501.00778
  Platzer, Sjodahl, Thoren 1808.00332
  Forshaw, Holguin, Platzer 1905.08686
  Isaacson, Prestel 1806.10102



- Electroweak corrections $\alpha/\alpha_s \sim 10\%$

  Christiansen, Sjostrand arXiv:1401.5238
  Krauss, Petrov, Schoenherr, Spannowsky arXiv:1403.4788
  Chen, Han, Tweedie arXiv:1611.00788
  Kleiss, RV 2002.09248

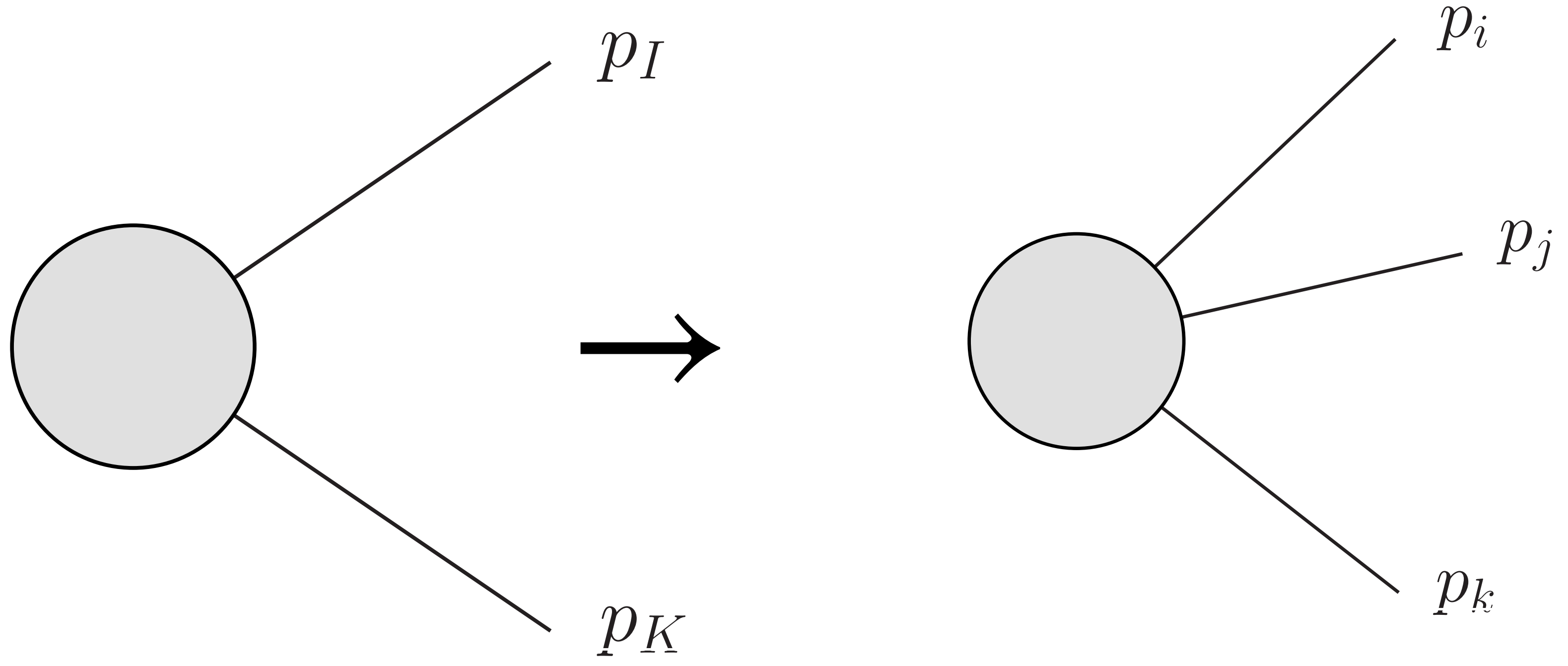
  → Rest of the talk

Vincia: Three ingredients

$$s_{ab} = 2p_a \cdot p_b$$
$$m_{ab}^2 = (p_a + p_b)^2$$

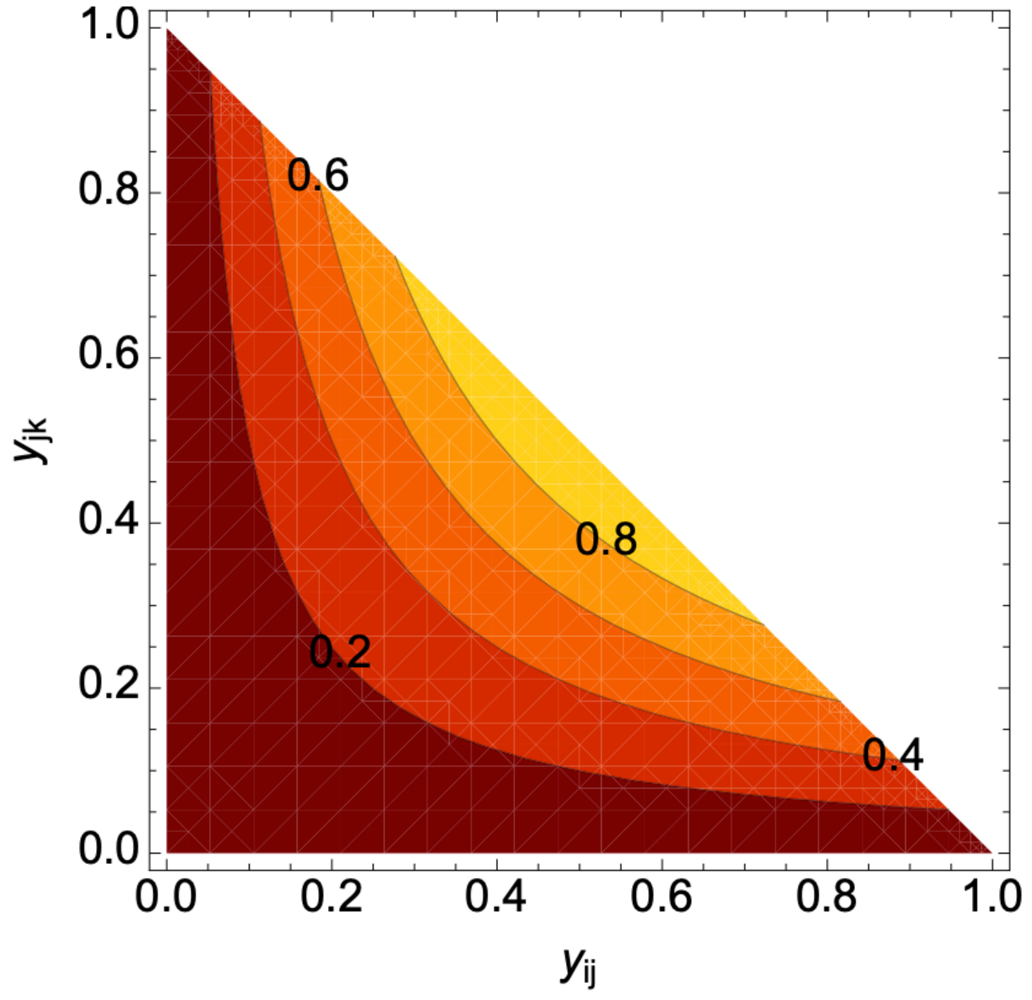
1. Phase space factorisation

$$d\Phi_{\text{ps}} = \frac{1}{16\pi^2} \lambda^{\frac{1}{2}}(m_{IK}^2, m_I^2, m_K^2)\, ds_{ij}\, ds_{jk} \frac{d\varphi}{2\pi}$$



2. Ordering scale: Ariadne $p_\perp^2$

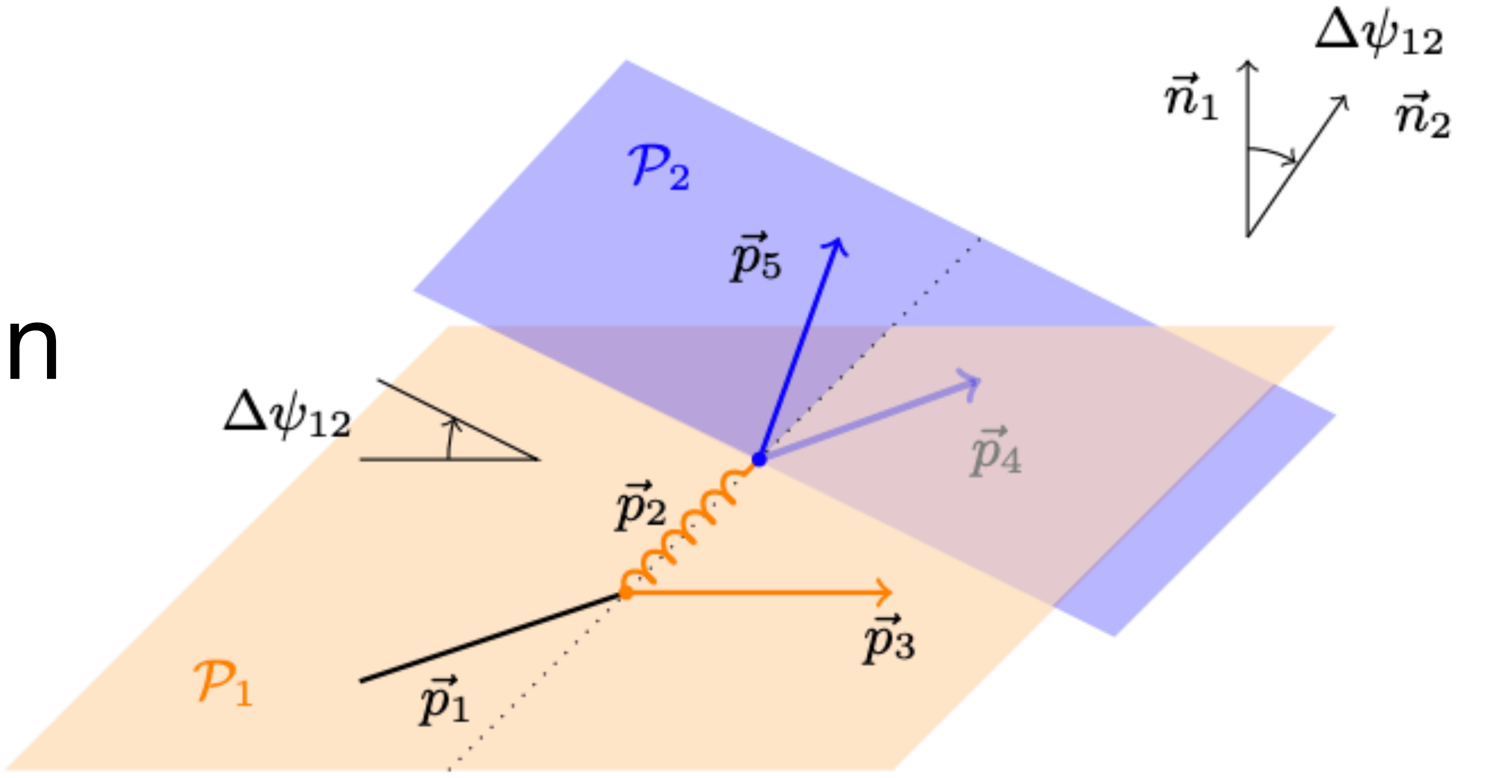
$$p_\perp^2 = \frac{s_{ij} s_{jk}}{s_{IK}}$$



3. Branching kernel: Antenna functions

$$a_{q\bar{q}}(s_{ij}, s_{jk}) = 4\pi\alpha_s C_F \left(2\frac{s_{ik}}{s_{ij}s_{jk}} - 2\frac{m_i^2}{s_{ij}^2} - 2\frac{m_k^2}{s_{jk}^2} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right)$$

Side note: Spin interference

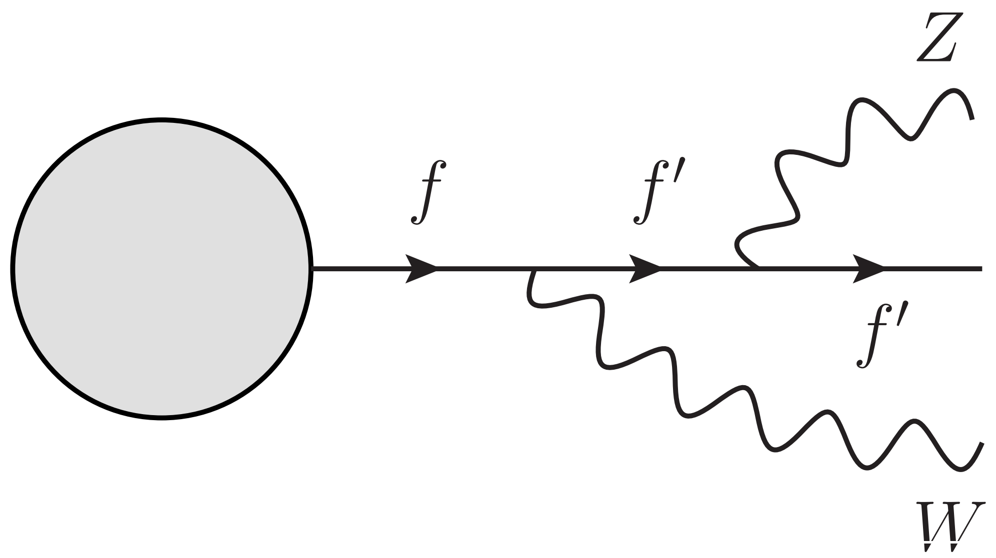


- In QCD, spin interference effects only lead to azimuthal modulation
  $\longrightarrow$ Integrates out of the Sudakov

$$\Delta(p_{\perp,i-1}^2, p_{\perp,i}^2) = \exp\left(-\int_{p_{\perp,i}^2}^{p_{\perp,i-1}^2} \underset{\big\uparrow}{d\Phi_{\text{ps}}} B(\Phi_{\text{ps}})\right)$$

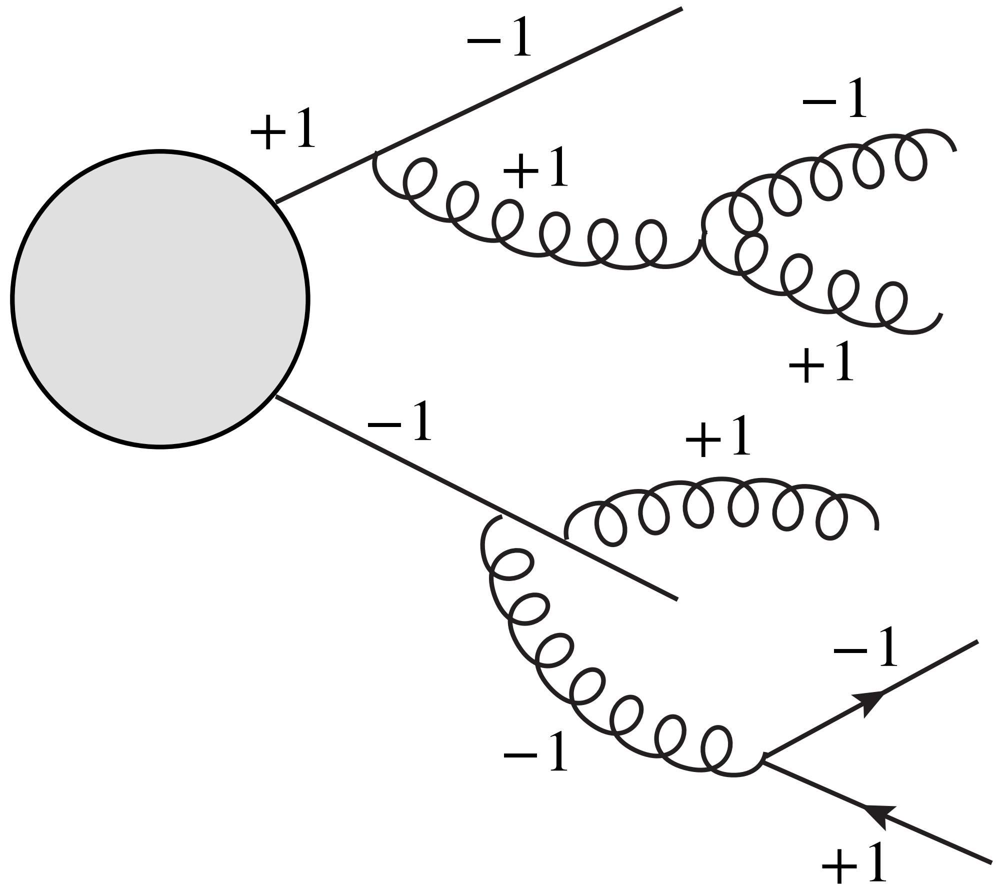
Azimuthal integral

- In EW, spin influences the rate of emissions
  $\longrightarrow$ Does not integrate out of the Sudakov



Vincia's solution: Evolution of intermediate helicity states

- Should capture leading effects
- Needs separate branching kernels for every spin configuration



Electroweak Showering

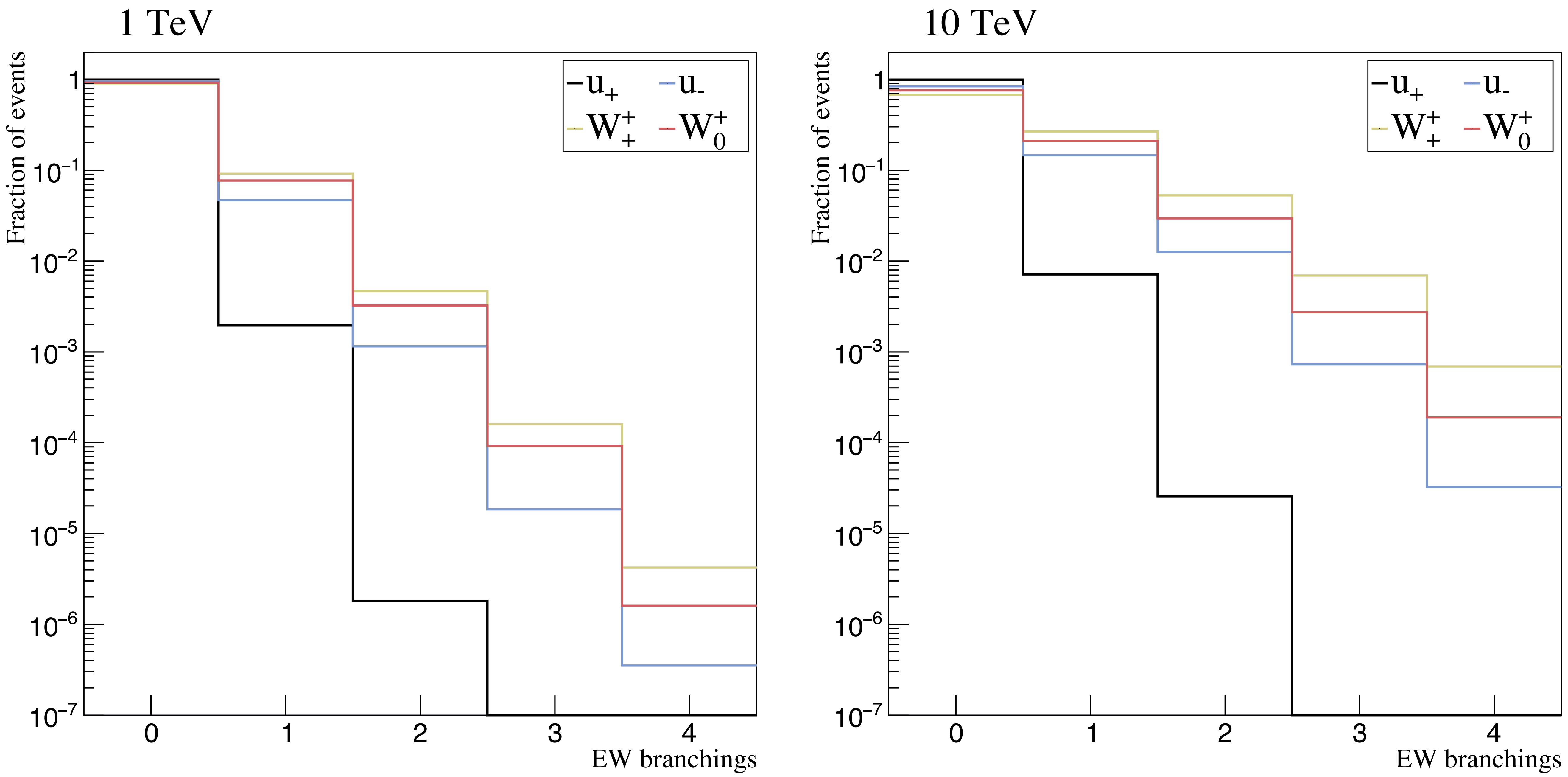
Why EW Showers?

- **Real corrections: EW gauge bosons, tops, Higgs part of jets**
- **Virtual corrections: Universal incorporation of Sudakov logs** $\frac{\alpha}{\pi}\ln^2\left(s/Q_{\mathrm{EW}}^2\right)$

Applications

- (HL)-LHC ATLAS 1609.07045
- Future colliders
- DM spectra Bauer, Rodd, Webber 2007.15001

⟶ Results later



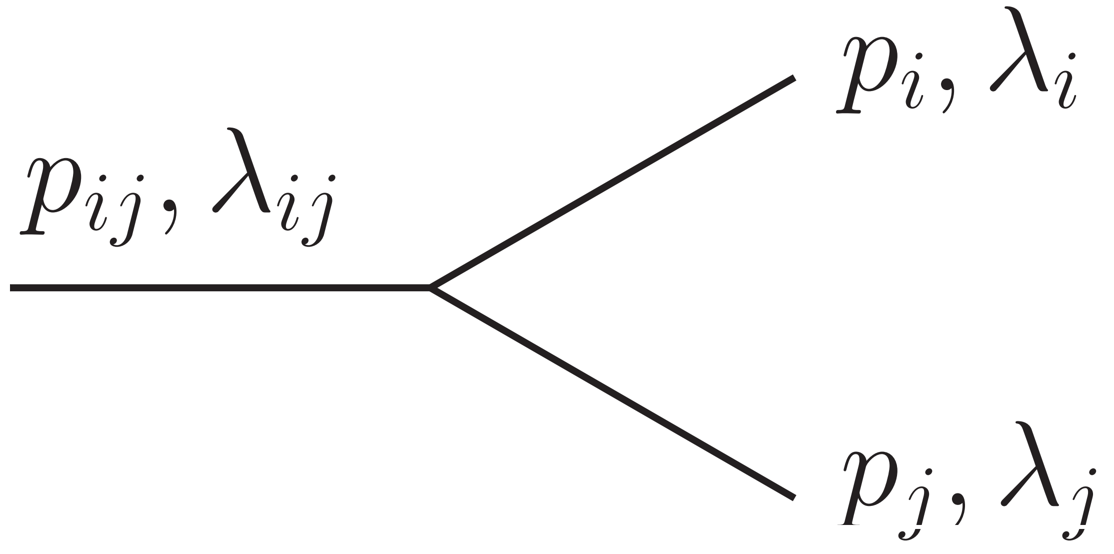
Existing implementations

- Only vector boson emissions Christiansen, Sjostrand arXiv:1401.5238
  Krauss, Petrov, Schoenherr, Spannowsky arXiv:1403.4788
- Full-fledged EW shower Chen, Han, Tweedie arXiv:1611.00788

Electroweak Branching Kernels

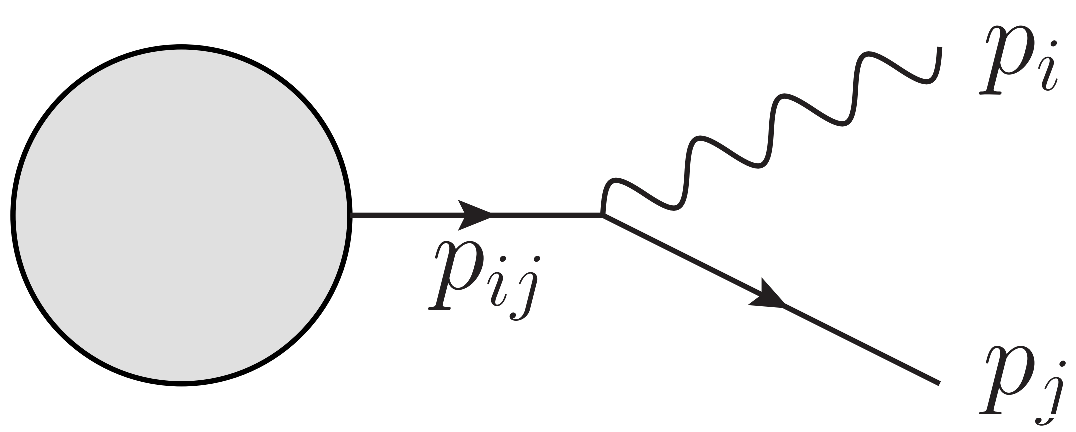
Use spinor-helicity formalism

$$M_{\lambda_{ij},\lambda_i,\lambda_j}(p_i,p_j) =$$



Transform to Vincia phase space

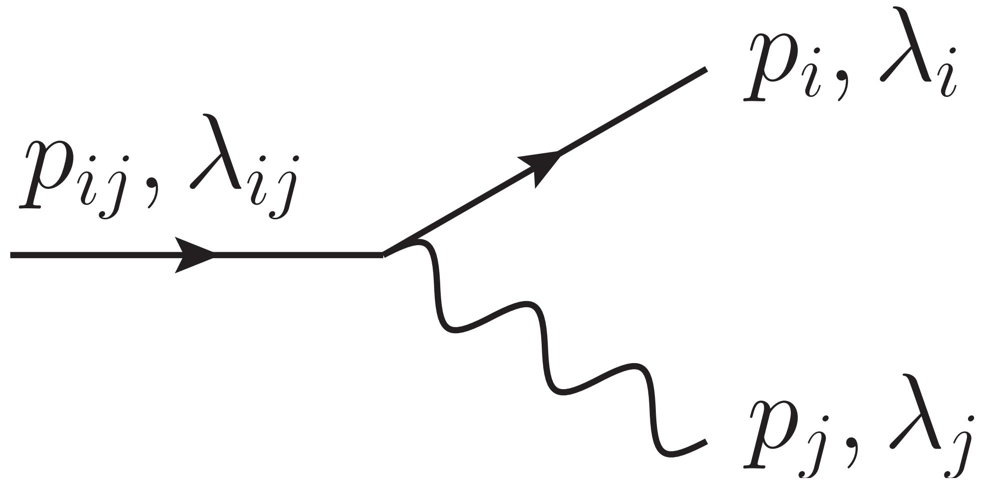
$$a_{\lambda_{ij},\lambda_i,\lambda_j}(s_{ij},s_{jk}) = \left[\left|\frac{1}{Q^2}M_{\lambda_{ij},\lambda_i,\lambda_j}(p_i,p_j)\right|^2\right]^{z\to x_i}_{(1-z)\to x_j}$$



$$x_i = \frac{s_{ij}+s_{ik}+m_i^2}{m_{IK}^2} \qquad x_j = \frac{s_{ij}+s_{jk}+m_j^2}{m_{IK}^2}$$

$$Q^2 = s_{ij}+m_i^2+m_j^2-m_{ij}^2$$

Longitudinal Polarisations



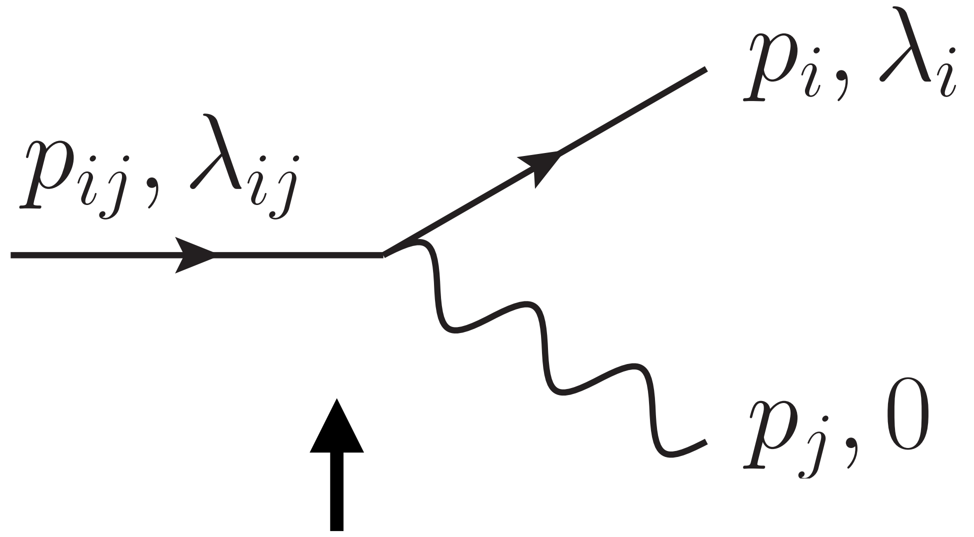
$$M_{\lambda_{ij},\lambda_i,\lambda_j}(p_i, p_j) = \bar{u}_{\lambda_i}(p_i)(v + a\gamma^5)\not{\epsilon}_{\lambda_j}(p_j)u_{\lambda_{ij}}(p_{ij})$$

1. Insert spinor representations
2. Consider longitudinal polarisation
3. Do some Dirac algebra

$$M_{+,+,0}(p_i, p_j) \propto \frac{1}{m_j}\left((Q^2 + m_{ij}^2)\not{p}_{ij} - m_i^2\not{p}_{ij}\right)$$

Q^2 drops out → Unitarity violation → ???

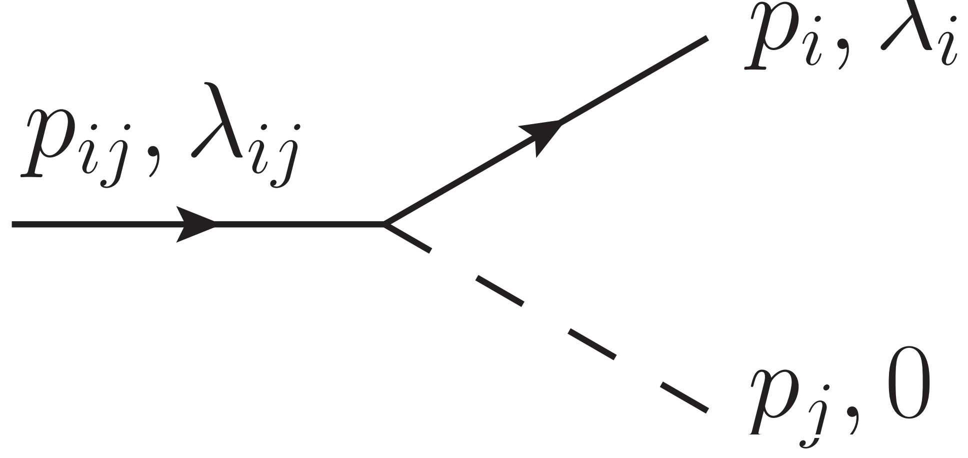
Goldstone Bosons



↑

$$\epsilon_0^\mu(p) = \frac{1}{m}\left(p^\mu - \frac{m^2}{p\cdot k}k^\mu\right)$$

↓



Goldstone piece actually couples to Yukawa

Possible to solve with Goldstone equivalence and suitable gauge choice

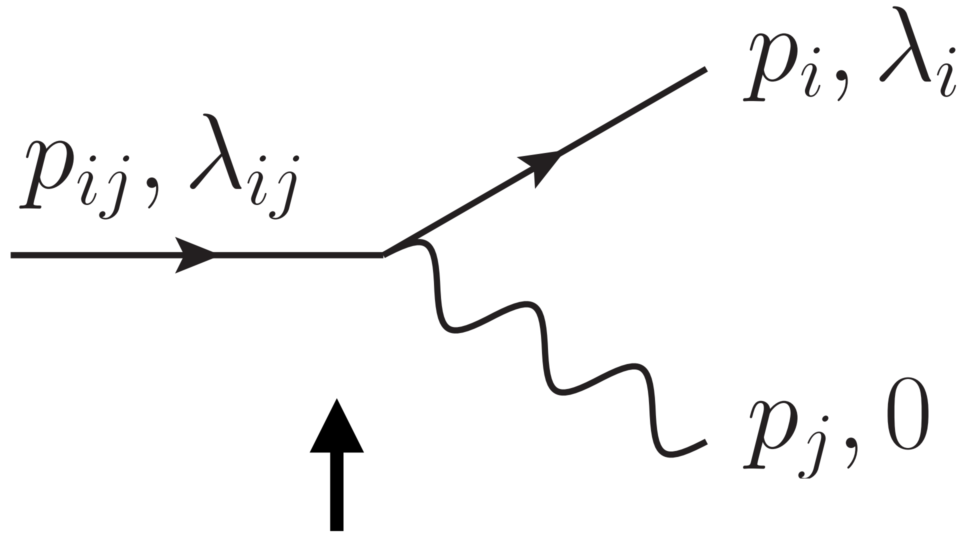
Spinor helicity formalism enables much simpler solution:

Yukawa couplings

$$\frac{1}{m_j}\left(\left(\underset{\downarrow}{Q^2} + \overset{\uparrow}{m_{ij}^2}\right)\not{p}_i - \overset{\uparrow}{m_i^2}\not{p}_{ij}\right)$$

Off-shellness

Goldstone Bosons



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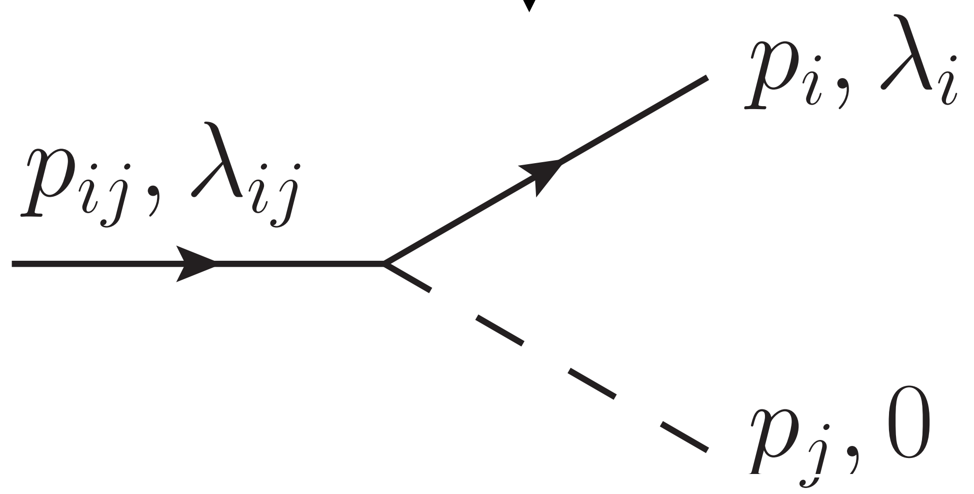

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Possible to solve with Goldstone equivalence and suitable gauge choice

Spinor helicity formalism enables much simpler solution:

Yukawa couplings

$$\frac{1}{m_j}\left((\cancel{Q}^2 + m_{ij}^2)\not{p}_i - m_i^2 \not{p}_{ij}\right)$$

Off-shellness

Collinear Limits

λ_I	λ_i	λ_j	$V \to f\bar{f}'$
λ	λ	$-\lambda$	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\, z$
λ	$-\lambda$	λ	$\sqrt{2}\lambda(v+\lambda a)\sqrt{\tilde{Q}^2}\,(1-z)$
λ	λ	λ	$\sqrt{2}\lambda\left[m_i(v+\lambda a)\sqrt{\frac{1-z}{z}} + m_j(v-\lambda a)\sqrt{\frac{z}{1-z}}\right]$
λ	$-\lambda$	$-\lambda$	0
0	λ	λ	$\sqrt{\tilde{Q}^2}\left[\frac{m_i}{m_{ij}}(v+\lambda a) + \frac{m_j}{m_{ij}}(v-\lambda a)\right]$
0	λ	$-\lambda$	$(v-\lambda a)\left[2m_{ij}\sqrt{z(1-z)} - \frac{m_i^2}{m_{ij}}\sqrt{\frac{1-z}{z}} - \frac{m_j^2}{m_{ij}}\sqrt{\frac{z}{1-z}}\right] + (v+\lambda a)\frac{m_i m_j}{m_{ij}}\frac{1}{\sqrt{z(1-z)}}$

λ_{ij}	λ_i	λ_j	$f \to f'V$ and $\bar{f} \to \bar{f}'V$
λ	λ	λ	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,\frac{1}{\sqrt{1-z}}$
λ	λ	$-\lambda$	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,\frac{z}{\sqrt{1-z}}$
λ	$-\lambda$	λ	$\sqrt{2}\lambda\left[m_{ij}(v-\lambda a)\sqrt{z} - m_i(v+\lambda a)\frac{1}{\sqrt{z}}\right]$
λ	$-\lambda$	$-\lambda$	0
λ	λ	0	$(v-\lambda a)\left[\frac{m_{ij}^2}{m_j}\sqrt{z} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{z}} - 2m_j\frac{\sqrt{z}}{1-z}\right] + (v+\lambda a)\frac{m_i m_{ij}}{m_j}\frac{1-z}{\sqrt{z}}$
λ	$-\lambda$	0	$\sqrt{\tilde{Q}^2}\sqrt{1-z}\left[\frac{m_i}{m_j}(v-\lambda a) - \frac{m_{ij}}{m_j}(v+\lambda a)\right]$

λ_I	λ_i	$(f \to fh$ and $\bar{f} \to \bar{f}h) \times \frac{e}{2s_w}\frac{m_f}{m_w}$
λ	λ	$m_f\left[\sqrt{z} + \frac{1}{\sqrt{z}}\right]$
λ	$-\lambda$	$\sqrt{1-z}\sqrt{\tilde{Q}^2}$

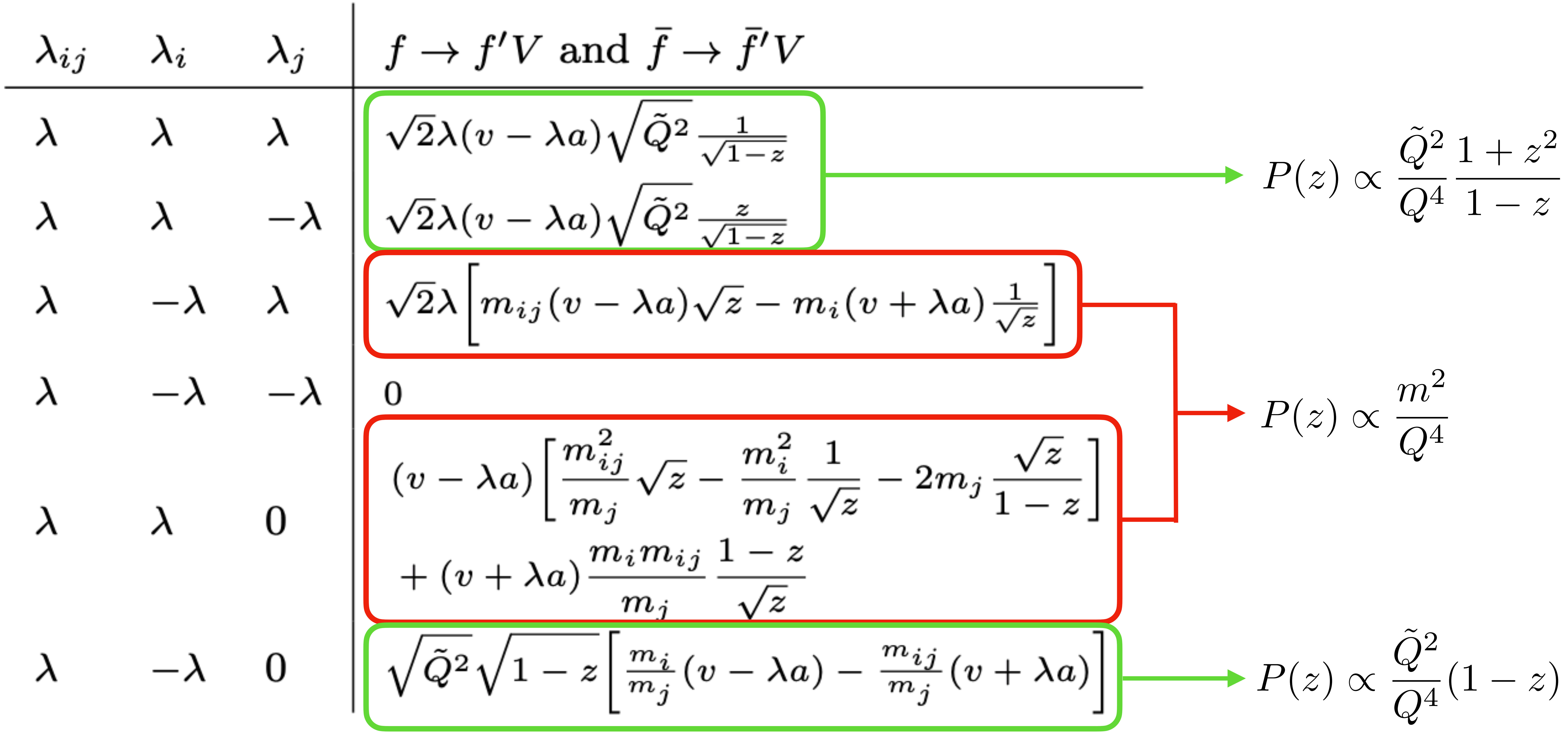
λ_I	λ_i	$V \to Vh \times g_h$
λ	λ	-1
λ	$-\lambda$	0
0	λ	$\frac{1}{m_{ij}}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{z(1-z)}$
λ	0	$\frac{1}{m_i}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
0	0	$\frac{1}{2}\frac{m_j^2}{m_i^2} + \frac{1-z}{z} + z$

λ_i	λ_i	$h \to VV \times g_V$
λ	λ	0
λ	$-\lambda$	-1
0	λ	$\frac{1}{m_i}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
λ	0	$\frac{1}{m_j}\frac{\lambda}{\sqrt{2}}\sqrt{\tilde{Q}^2}\sqrt{\frac{z}{1-z}}$
0	0	$\frac{1}{2}\frac{m_{ij}^2}{m_i^2} - 1 - \frac{1-z}{z} - \frac{z}{1-z}$

λ_i	λ_j	$h \to f\bar{f} \times \frac{e}{2s_w}\frac{m_f}{m_w}$
λ	λ	$\sqrt{\tilde{Q}^2}$
λ	$-\lambda$	$m_f\left[\sqrt{\frac{1-z}{z}} - \sqrt{\frac{z}{1-z}}\right]$

λ_I	λ_i	λ_j	$V \to V'V'' \times g_V$
λ	λ	λ	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}\sqrt{\frac{1}{z(1-z)}}$
λ	λ	$-\lambda$	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}z\sqrt{\frac{z}{1-z}}$
λ	$-\lambda$	λ	$\sqrt{2}\lambda\sqrt{\tilde{Q}^2}(1-z)\sqrt{\frac{1-z}{z}}$
λ	$-\lambda$	$-\lambda$	0
0	λ	λ	0
0	λ	$-\lambda$	$m_{ij}(2z-1) + \frac{m_j^2}{m_{ij}} - \frac{m_i^2}{m_{ij}}$
λ	0	λ	$m_i\left(1 + 2\frac{1-z}{z}\right) + \frac{m_j^2}{m_i} - \frac{m_{ij}^2}{m_i}$
λ	0	$-\lambda$	0
λ	λ	0	$m_j\left(1 + 2\frac{z}{1-z}\right) + \frac{m_i^2}{m_j} - \frac{m_{ij}^2}{m_j}$
λ	$-\lambda$	0	0
λ	0	0	$\frac{\lambda}{\sqrt{2}}\frac{m_i^2+m_j^2-m_{ij}^2}{m_i m_j}\sqrt{\tilde{Q}^2}\sqrt{z(1-z)}$
0	λ	0	$\frac{\lambda}{\sqrt{2}}\frac{m_{ij}^2+m_j^2-m_i^2}{m_{ij} m_j}\sqrt{\tilde{Q}^2}\sqrt{\frac{1-z}{z}}$
0	0	λ	$\frac{\lambda}{\sqrt{2}}\frac{m_{ij}^2+m_i^2-m_j^2}{m_{ij} m_i}\sqrt{\tilde{Q}^2}\sqrt{\frac{z}{1-z}}$
0	0	0	$\frac{1}{2}\frac{m_{ij}^3}{m_i m_j}(2z-1) - \frac{m_i^3}{m_{ij} m_j}\left(\frac{1}{2} + \frac{1-z}{z}\right) + \frac{m_j^3}{m_{ij} m_i}\left(\frac{1}{2} + \frac{z}{1-z}\right) + \frac{m_i m_j}{m_{ij}}\left(\frac{1-z}{z} - \frac{z}{1-z}\right) + \frac{m_{ij} m_i}{m_j}(1-z)\left(2 + \frac{1-z}{z}\right) - \frac{m_{ij} m_j}{m_i}z\left(2 + \frac{z}{1-z}\right)$

Collinear Limits



λ_{ij}	λ_i	λ_j	$f \to f'V$ and $\bar{f} \to \bar{f}'V$
λ	λ	λ	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,\frac{1}{\sqrt{1-z}}$
λ	λ	$-\lambda$	$\sqrt{2}\lambda(v-\lambda a)\sqrt{\tilde{Q}^2}\,\frac{z}{\sqrt{1-z}}$
λ	$-\lambda$	λ	$\sqrt{2}\lambda\left[m_{ij}(v-\lambda a)\sqrt{z} - m_i(v+\lambda a)\frac{1}{\sqrt{z}}\right]$
λ	$-\lambda$	$-\lambda$	0
λ	λ	0	$(v-\lambda a)\left[\frac{m_{ij}^2}{m_j}\sqrt{z} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{z}} - 2m_j\frac{\sqrt{z}}{1-z}\right] + (v+\lambda a)\frac{m_i m_{ij}}{m_j}\frac{1-z}{\sqrt{z}}$
λ	$-\lambda$	0	$\sqrt{\tilde{Q}^2}\sqrt{1-z}\left[\frac{m_i}{m_j}(v-\lambda a) - \frac{m_{ij}}{m_j}(v+\lambda a)\right]$

$$P(z) \propto \frac{\tilde{Q}^2}{Q^4}\frac{1+z^2}{1-z}$$

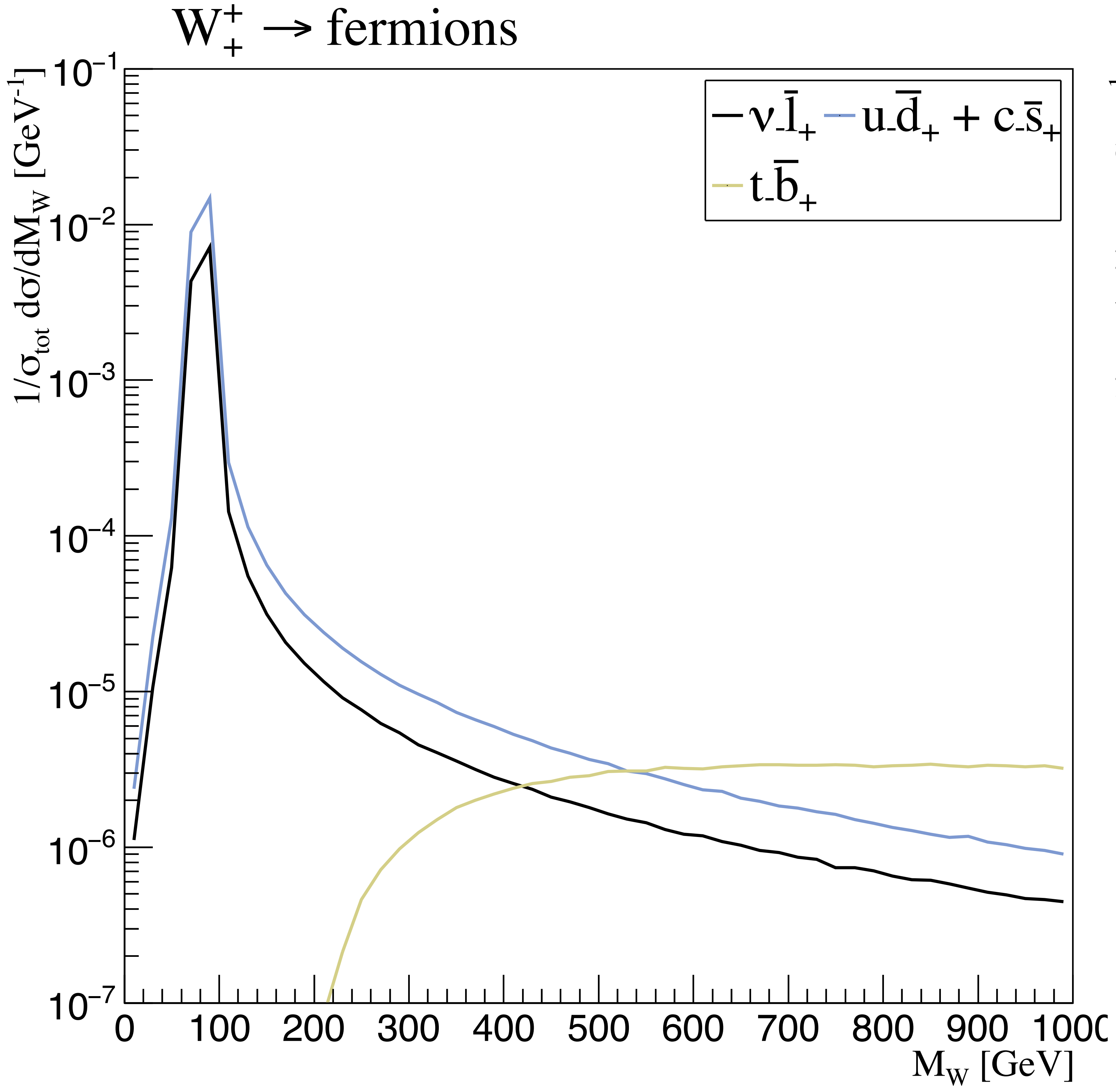
$$P(z) \propto \frac{m^2}{Q^4}$$

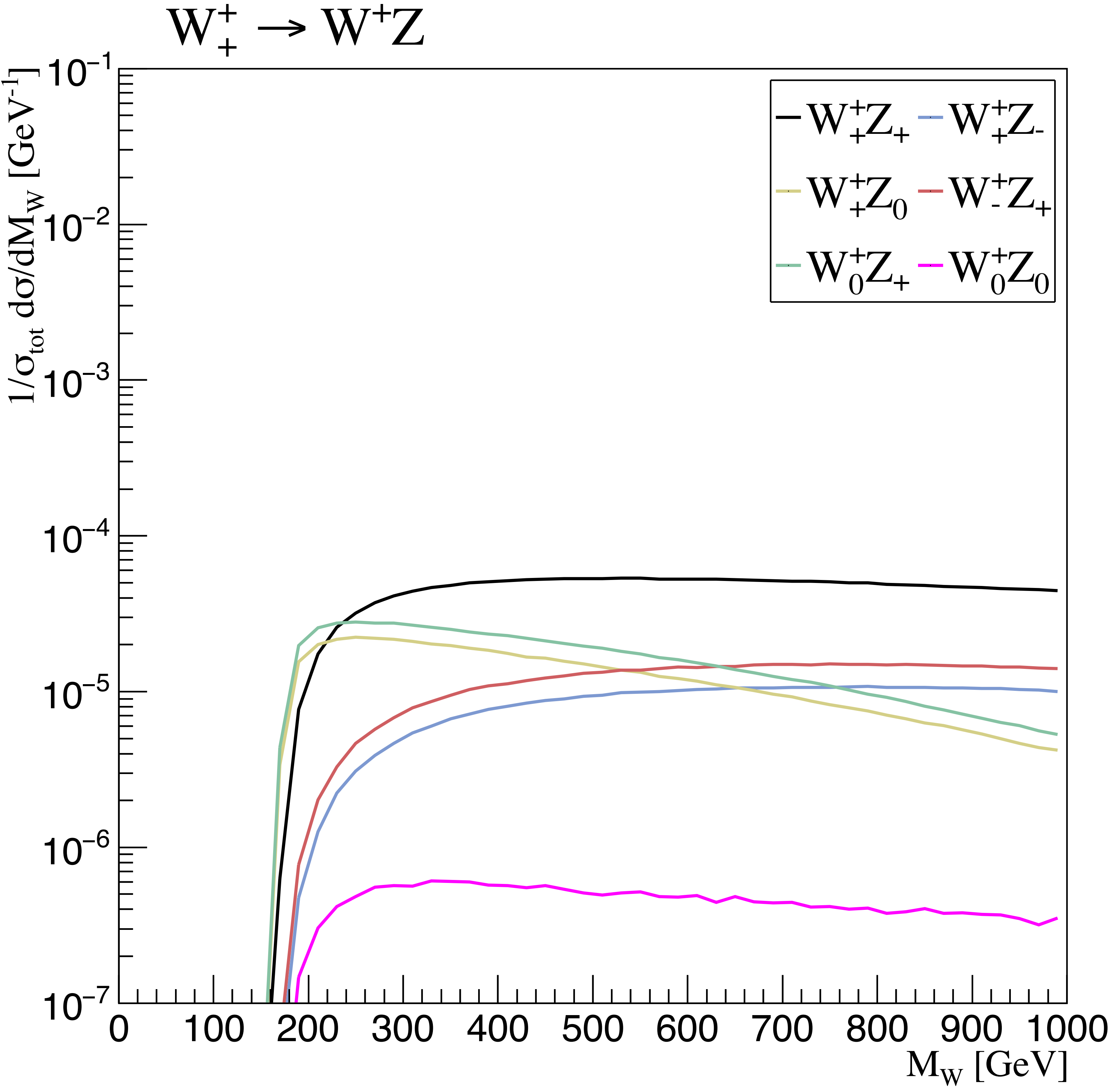
$$P(z) \propto \frac{\tilde{Q}^2}{Q^4}(1-z)$$

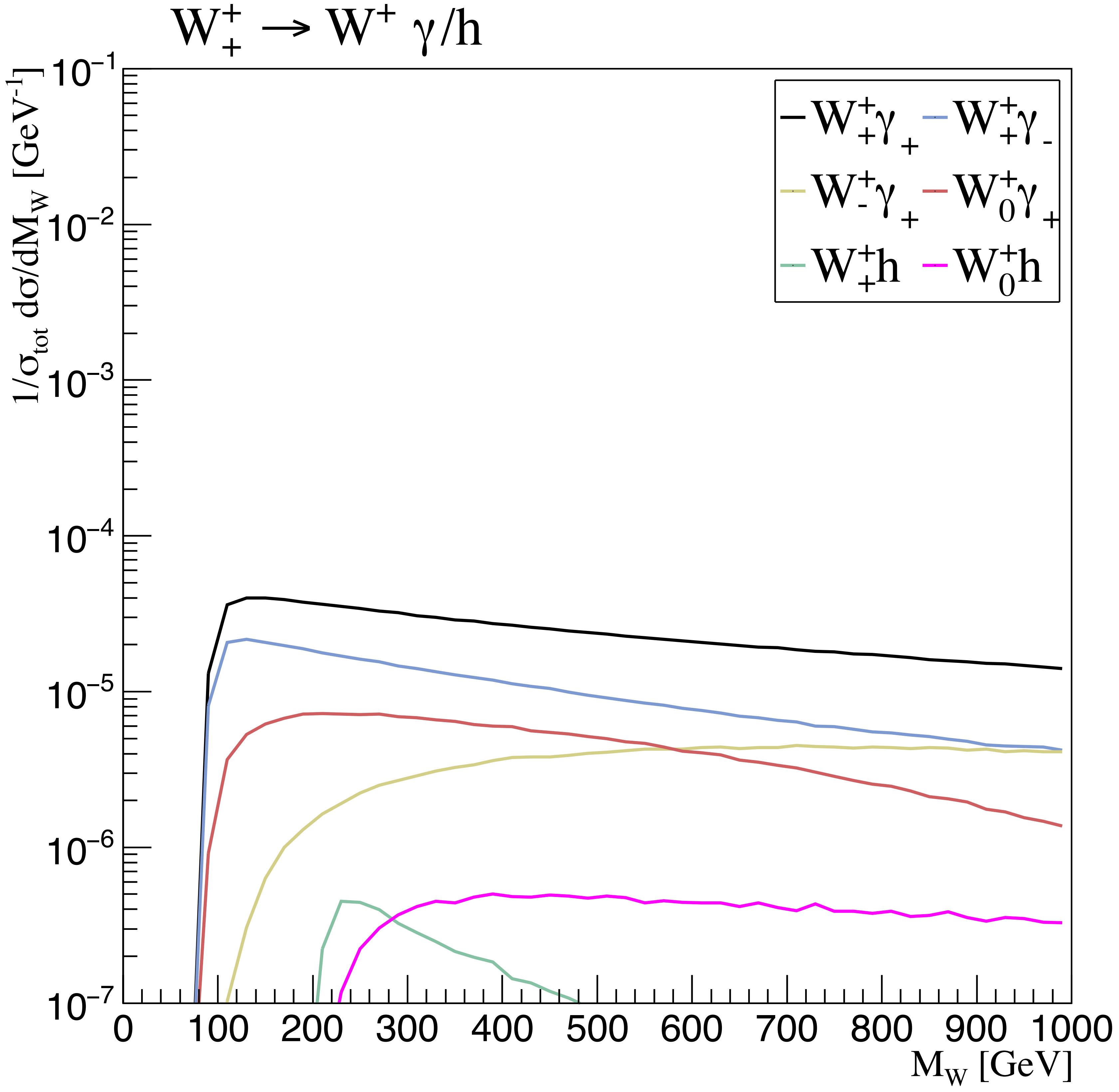
$$\tilde{Q}^2 = Q^2 + m_{ij}^2 - \frac{m_i^2}{z} - \frac{m_j^2}{1-z}$$

The Electroweak Shower

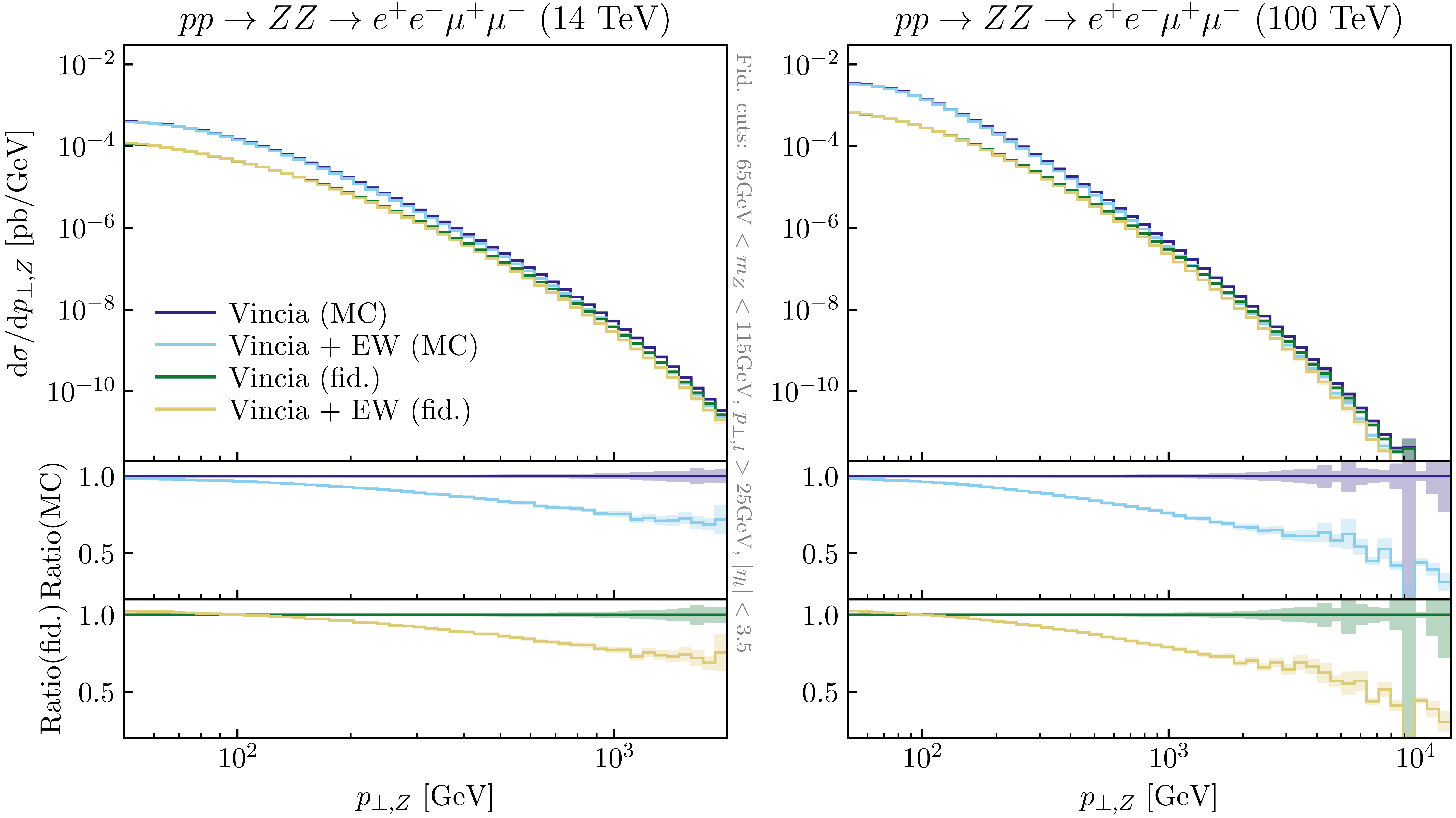
$\mathcal{O}(1000)$ types of branchings (all FSR + ffV ISR)







Results: Virtual Sudakov logs

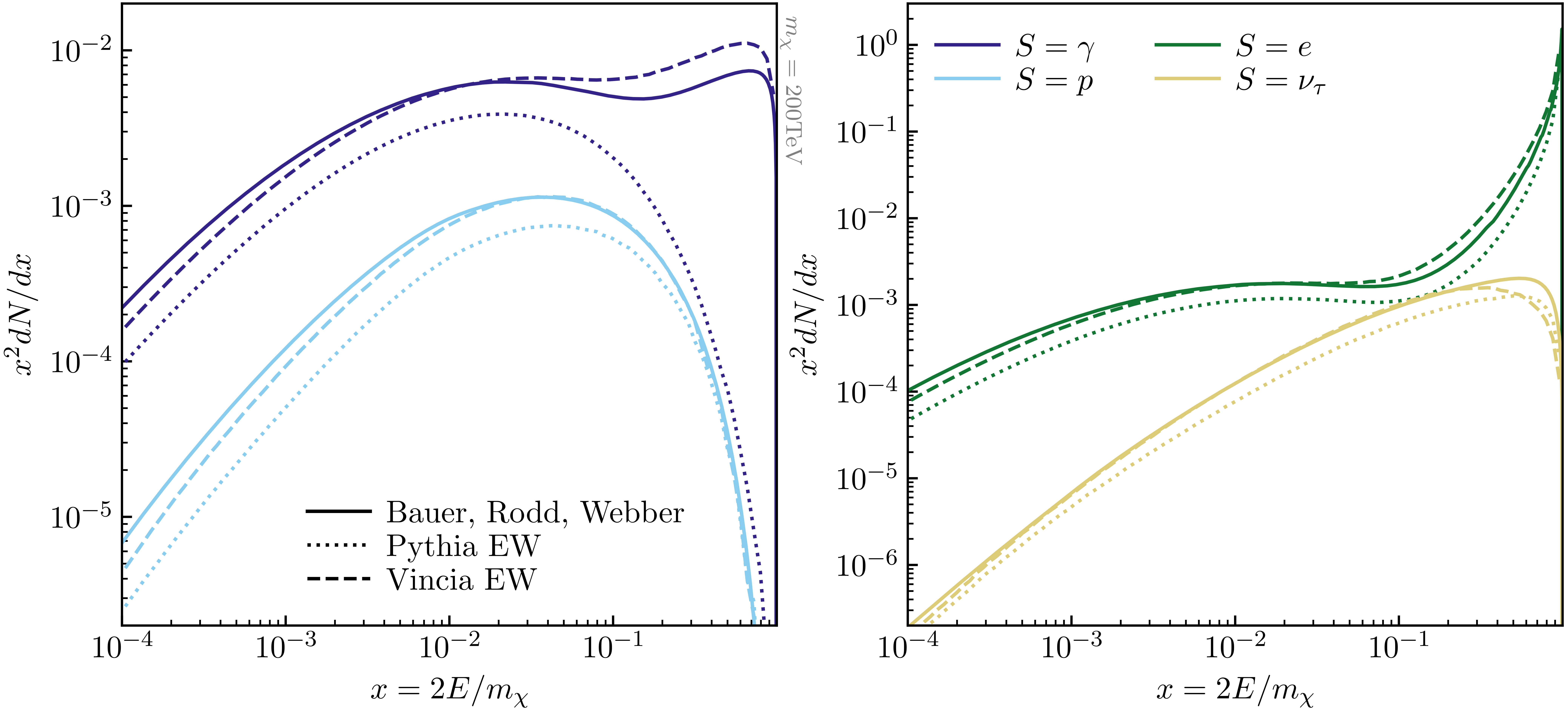


Results: DM decay spectra

Comparison with analytic results

Bauer, Rodd, Webber 2007.15001

$\chi \to \nu_e \bar{\nu}_e \to S$



Novel features in the Electroweak Sector

Resonance Matching

Branchings like $t \to bW$, $Z \to q\bar{q}$ etc.

- Large scales:
  EW shower offers best description
- Small scales:
  Breit-Wigner distribution

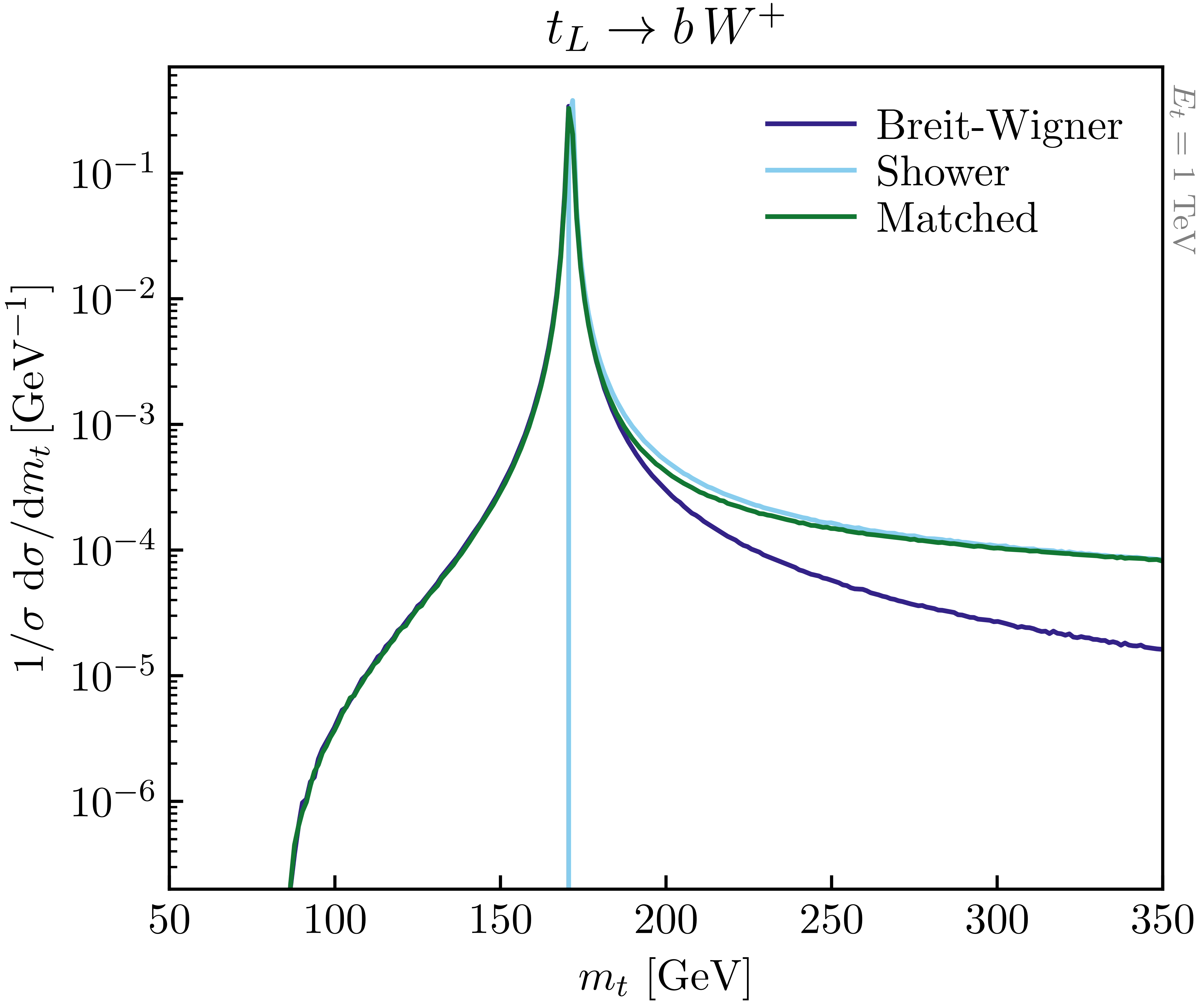
$$\mathrm{BW}(Q^2) \propto \frac{m_0\Gamma(m)}{Q^4 + m_0^2\Gamma(m)^2}$$

Matching:

- Sample mass from Breit-Wigner upon production
- Suppress shower by factor

$$\frac{Q^4}{(Q^2 + Q_{\mathrm{EW}}^2)^2}$$

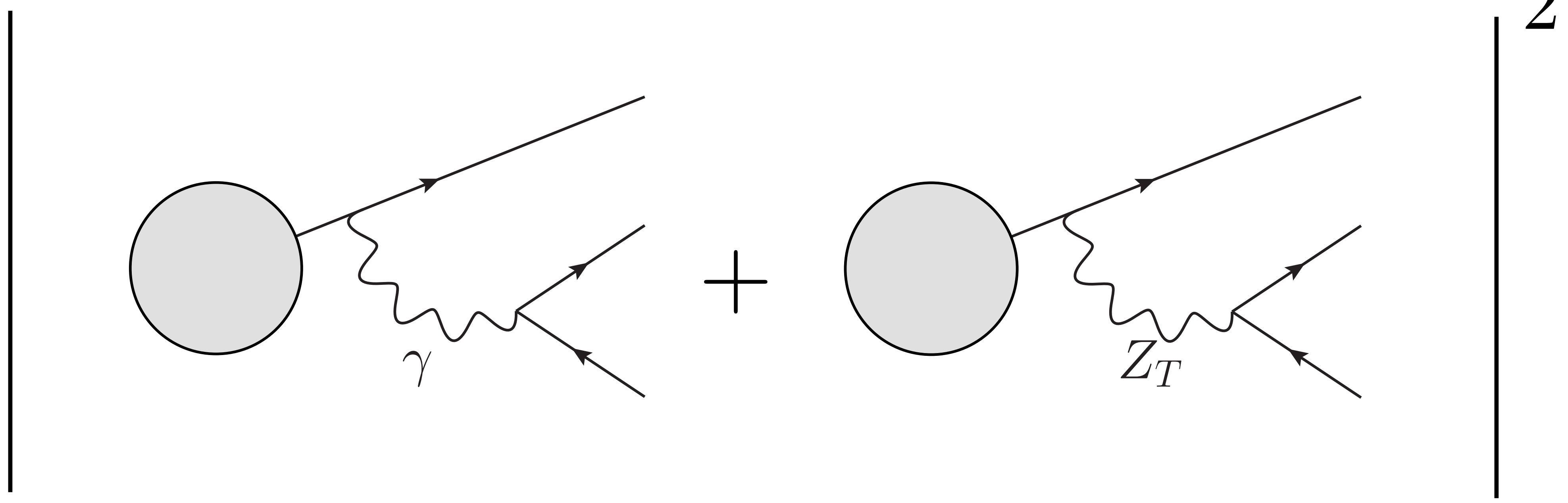
- Decay when shower hits off-shellness scale



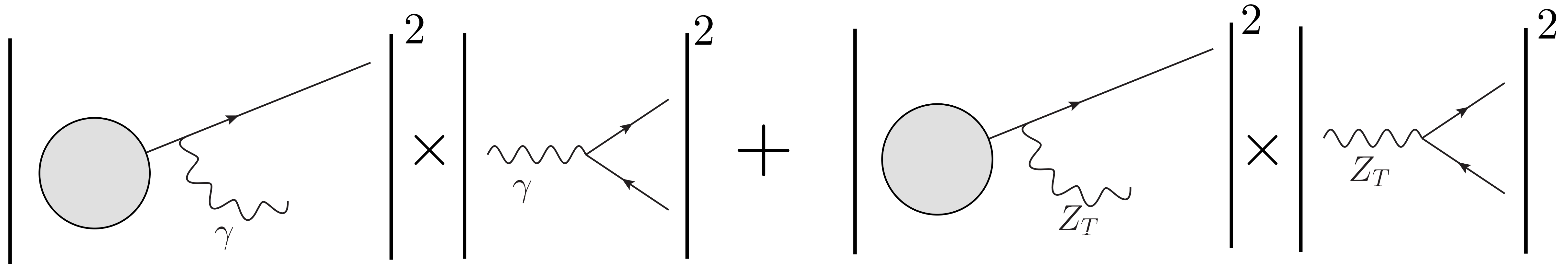
Neutral Boson Interference

Interference between γ, Z_T and h, Z_L

Physical contribution

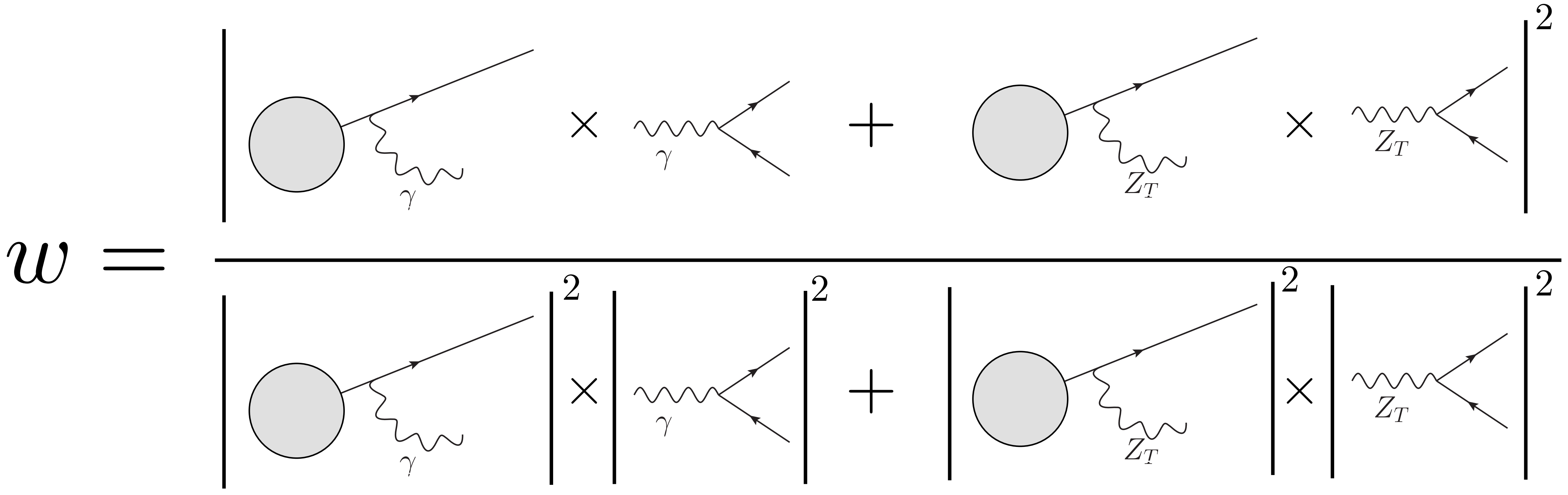


Shower approximation

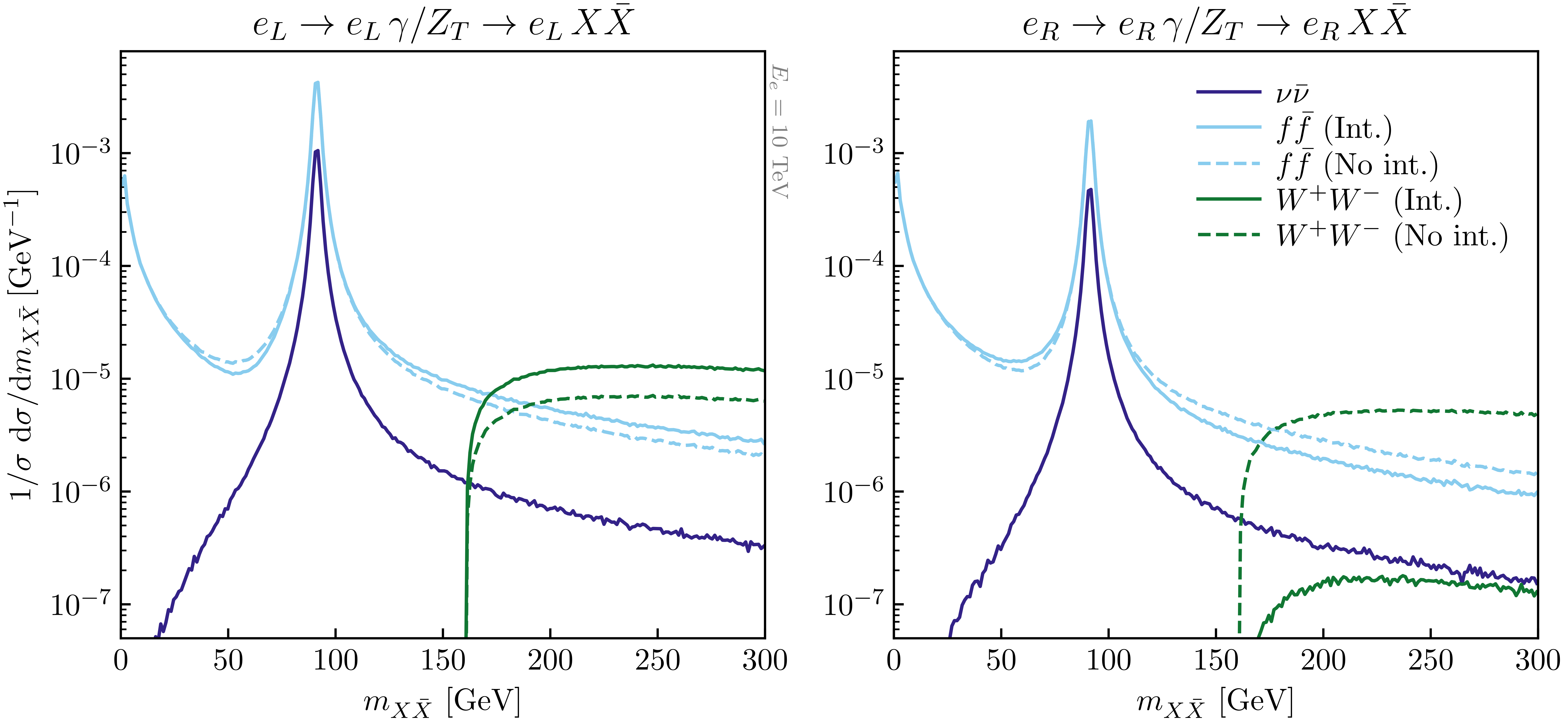


- Complicated solution: Evolve density matrices
  → Very computationally expensive

- Simple solution: Apply event weight
  → Does not get Sudakov right

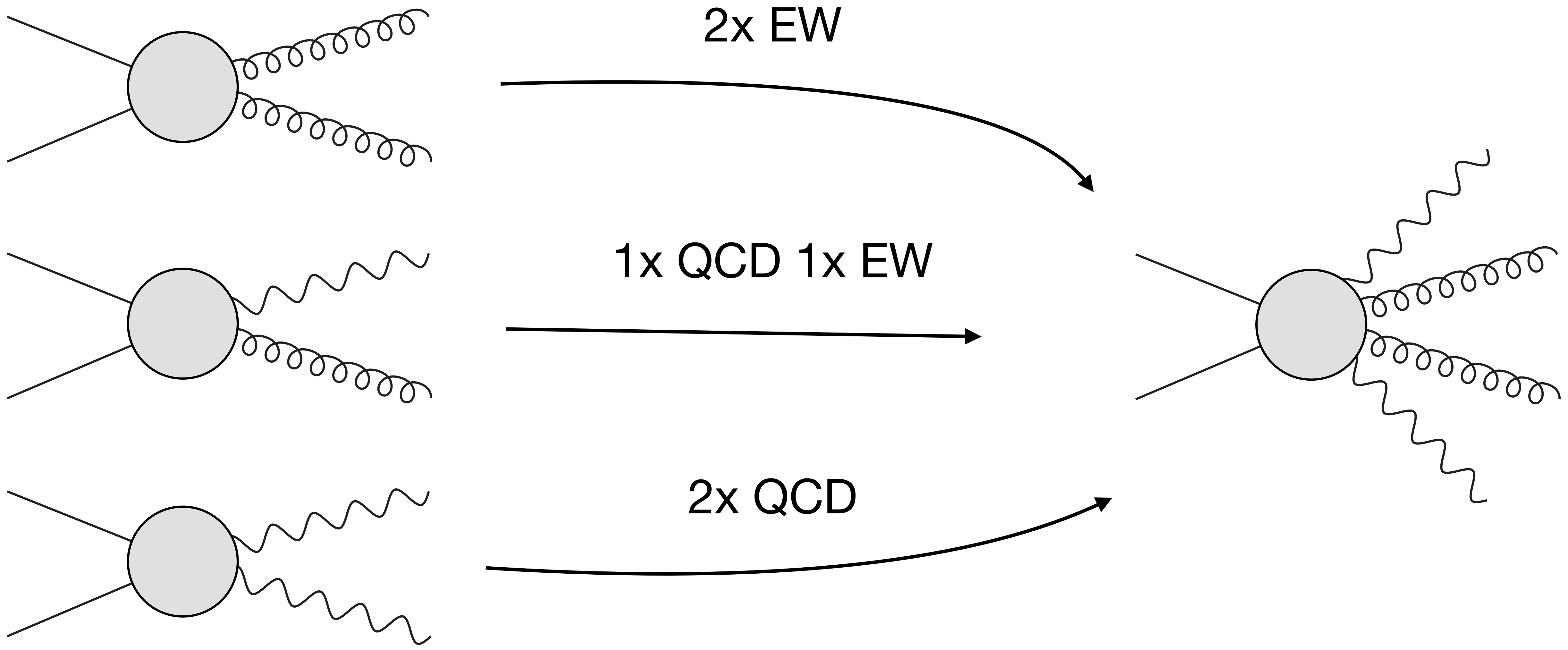


Bosonic Interference

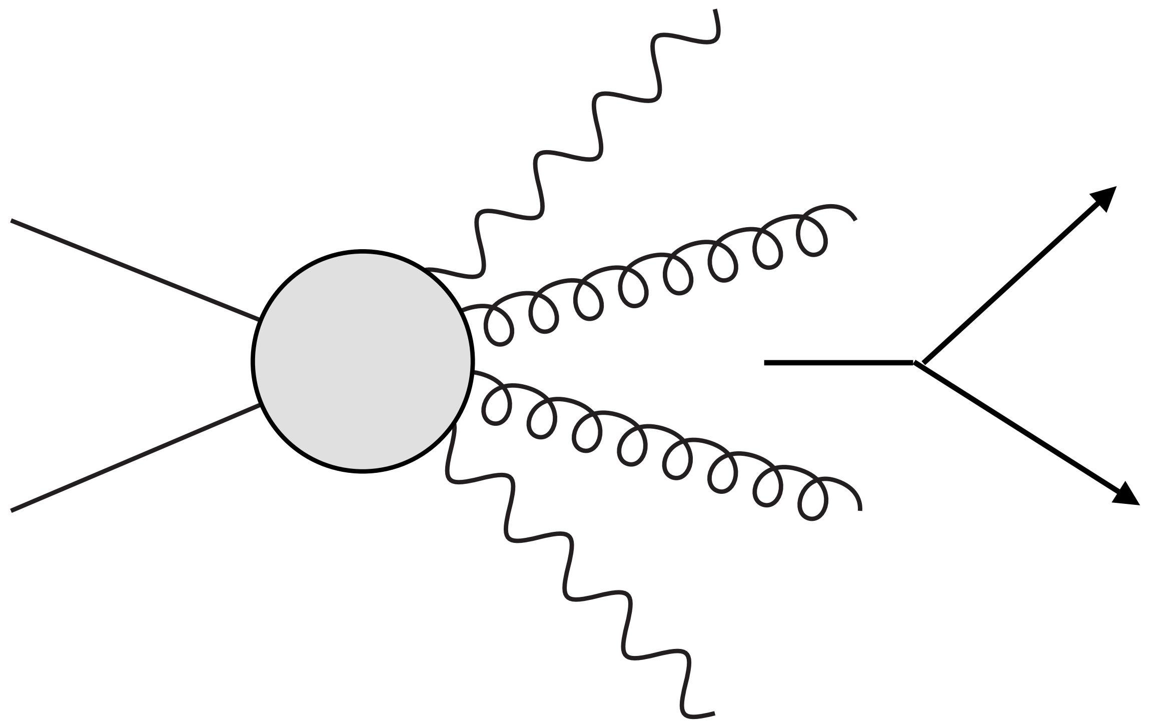


Overlap Veto

Double counting problem



Veto procedure



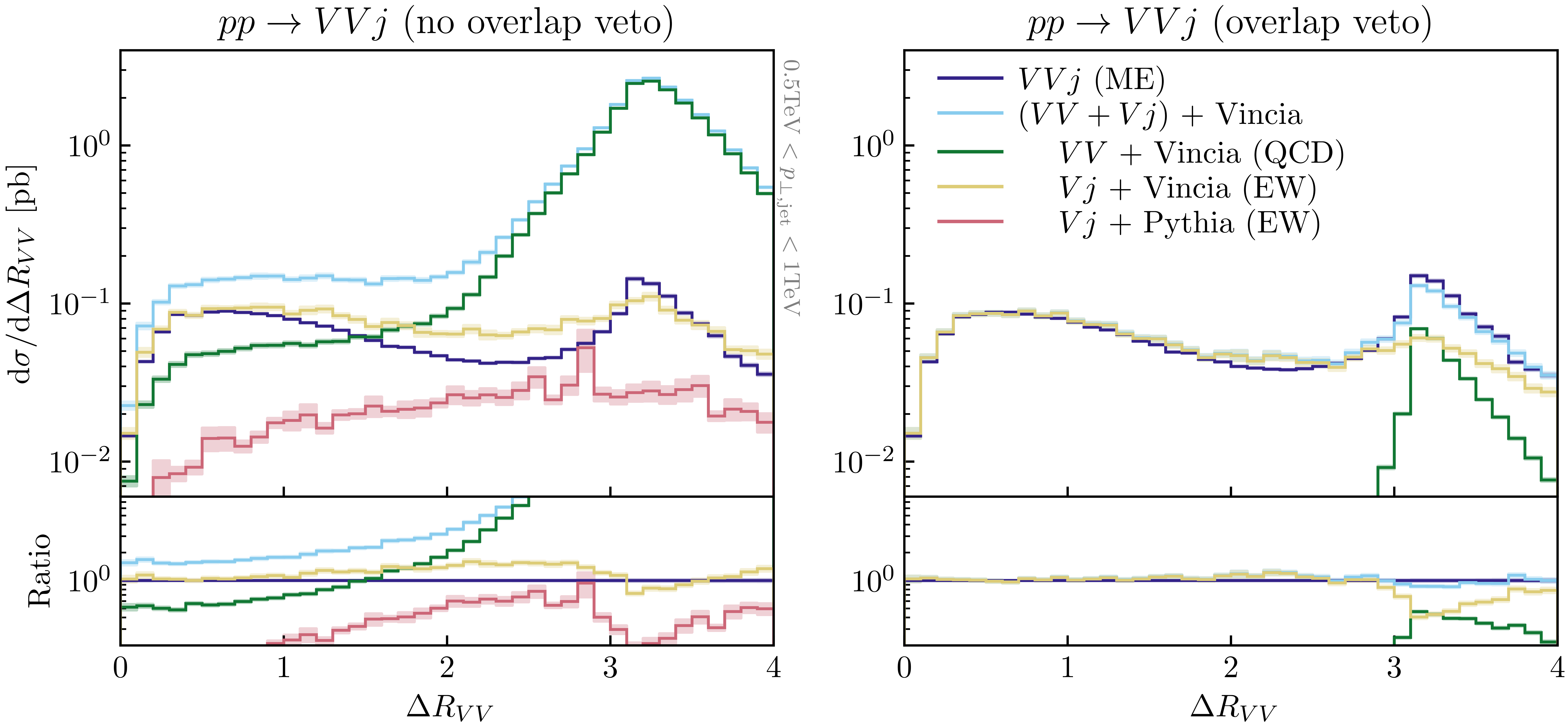
Last emission QCD? $\longrightarrow$ $d_{ij}^{\text{Last}} < \min\left(d_{ij}^{\text{EW}}\right)$

Last emission EW? $\longrightarrow$ $d_{ij}^{\text{Last}} < \min\left(d_{ij}^{\text{QCD}}\right)$

Accept Branching

$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right)\frac{\Delta_{ij}}{R} + m_i^2 + m_j^2 - m^2$$

Overlap Veto



Conclusions

- Universal EW radiative corrections relevant at (HL)-LHC and future colliders

- EW sector offers rich physics above the EW scale

- Many features unique to the EW sector
  - Matching to resonance decays
  - Neutral boson interference
  - Overlap between hard scatterings

- Many other features yet to implement
  - Treatment of soft & spin interference
  - Bloch-Nordsieck violations

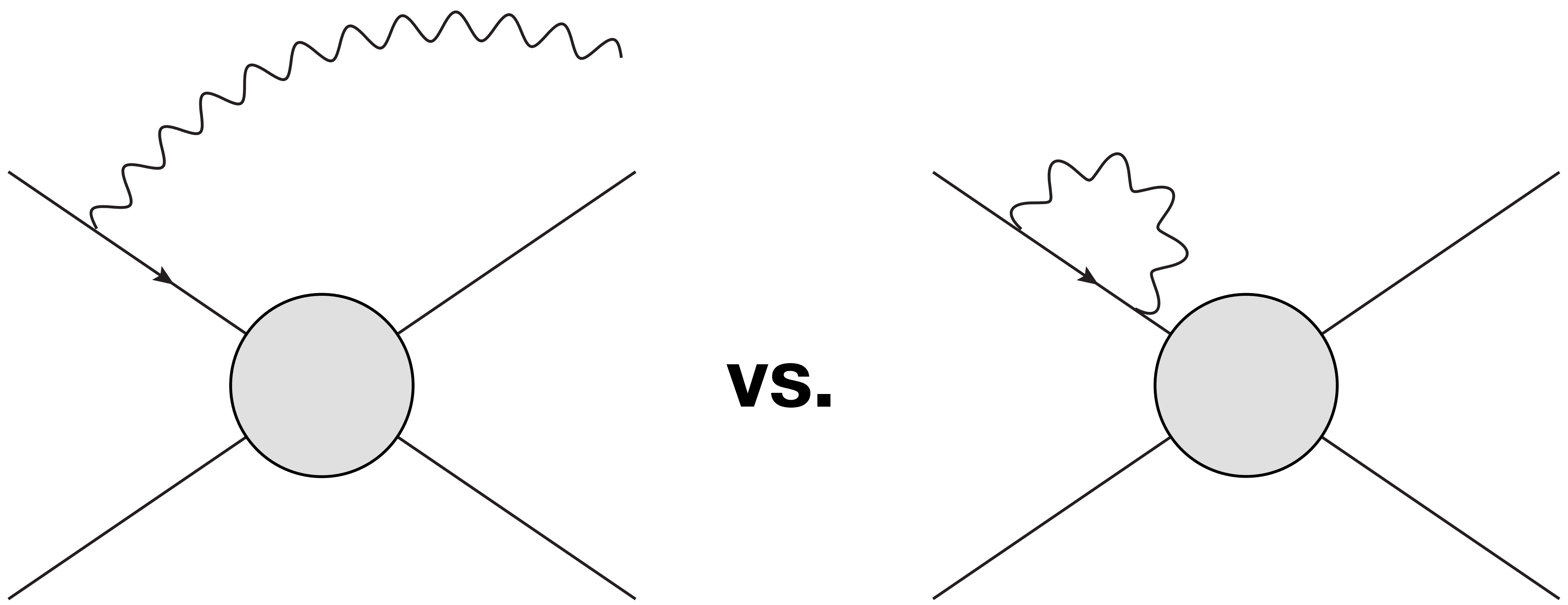
- EW shower is available in Pythia 8.304 (released last week)

Backup

Bloch-Nordsieck Violations

BN / KLN Theorems: Real and virtual IR singularities cancel

Requirement: Summing over gauge indices



W radiation in the initial state:
PDFs are not isospin symmetric
⟶ Incomplete cancellation

Effects not large at LHC, but will be significant at higher energies

No straightforward solution in shower language

Spinor-Helicity formalism

Fermion

$$u_\pm(p) = \frac{1}{\sqrt{2p\cdot k}}(\not{p} + m)u_\mp(k)$$

$$v_\pm(p) = \frac{1}{\sqrt{2p\cdot k}}(\not{p} - m)u_\mp(k)$$

$k \longrightarrow$ helicity for massive fermions

Gauge boson

$$\epsilon^\mu_\pm(p) = \pm\frac{1}{\sqrt{2}}\frac{1}{2p\cdot k}\bar{u}_\mp(k)\not{p}\gamma^\mu u_\pm(k)$$

$$\epsilon^\mu_0(p) = \frac{1}{m}\left(p^\mu - \frac{m^2}{p\cdot k}k^\mu\right)$$

$k \longrightarrow$ gauge choice

$$\boxed{k = (1, -\vec{e}_p)}$$

Spin points in direction of motion

Purely transverse & longitudinal

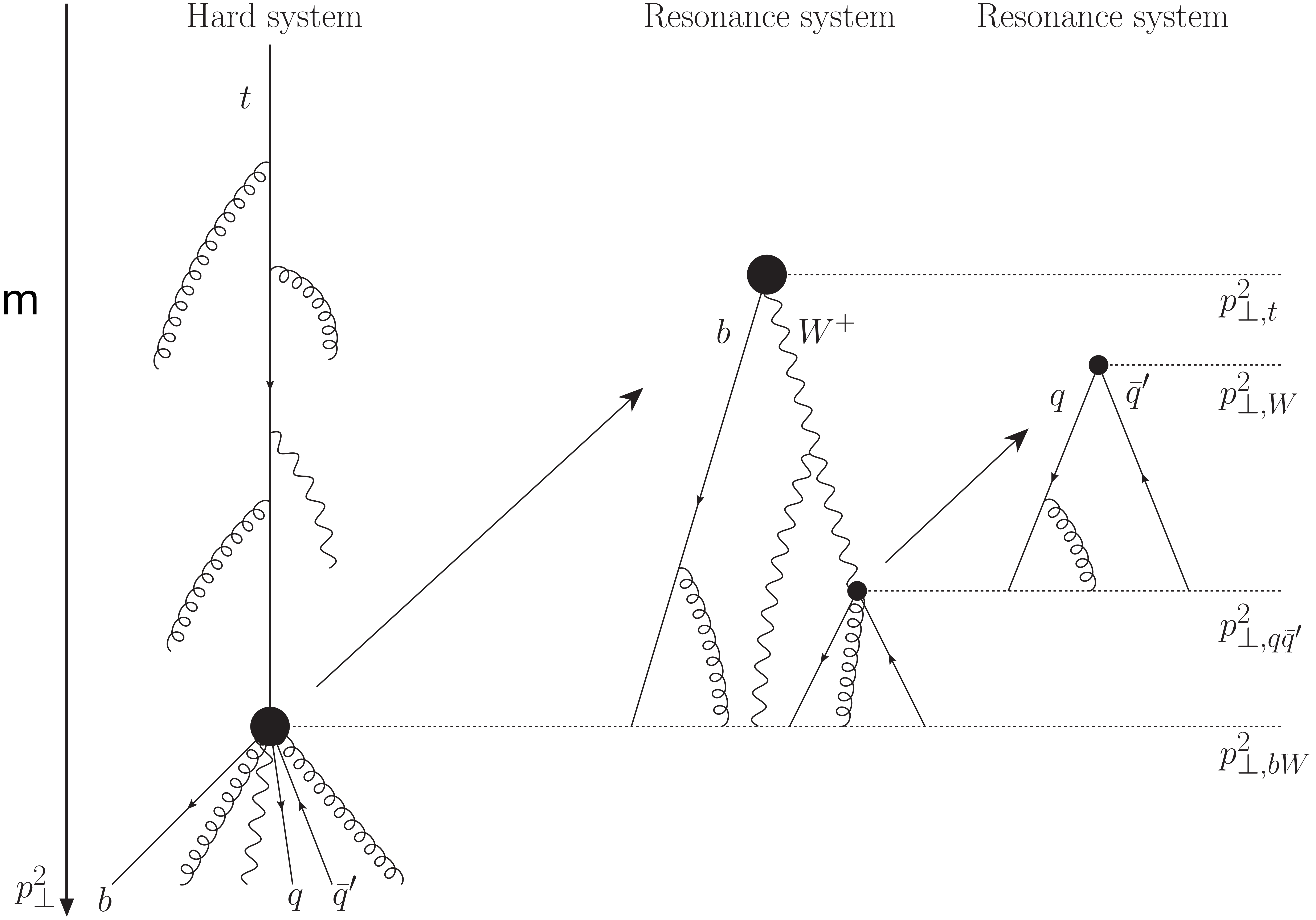
Resonance Matching

Pythia

- Narrow width approximation
- Decay showers after hard system

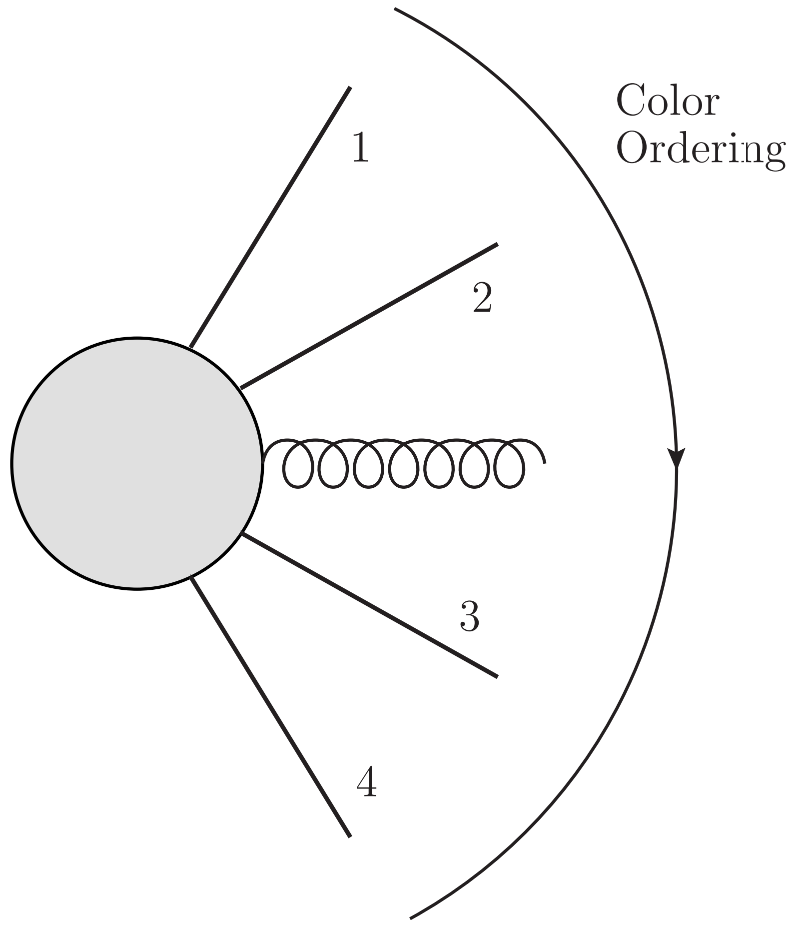
Vincia

- Decays part of hard system
- Natural treatment of finite width effects



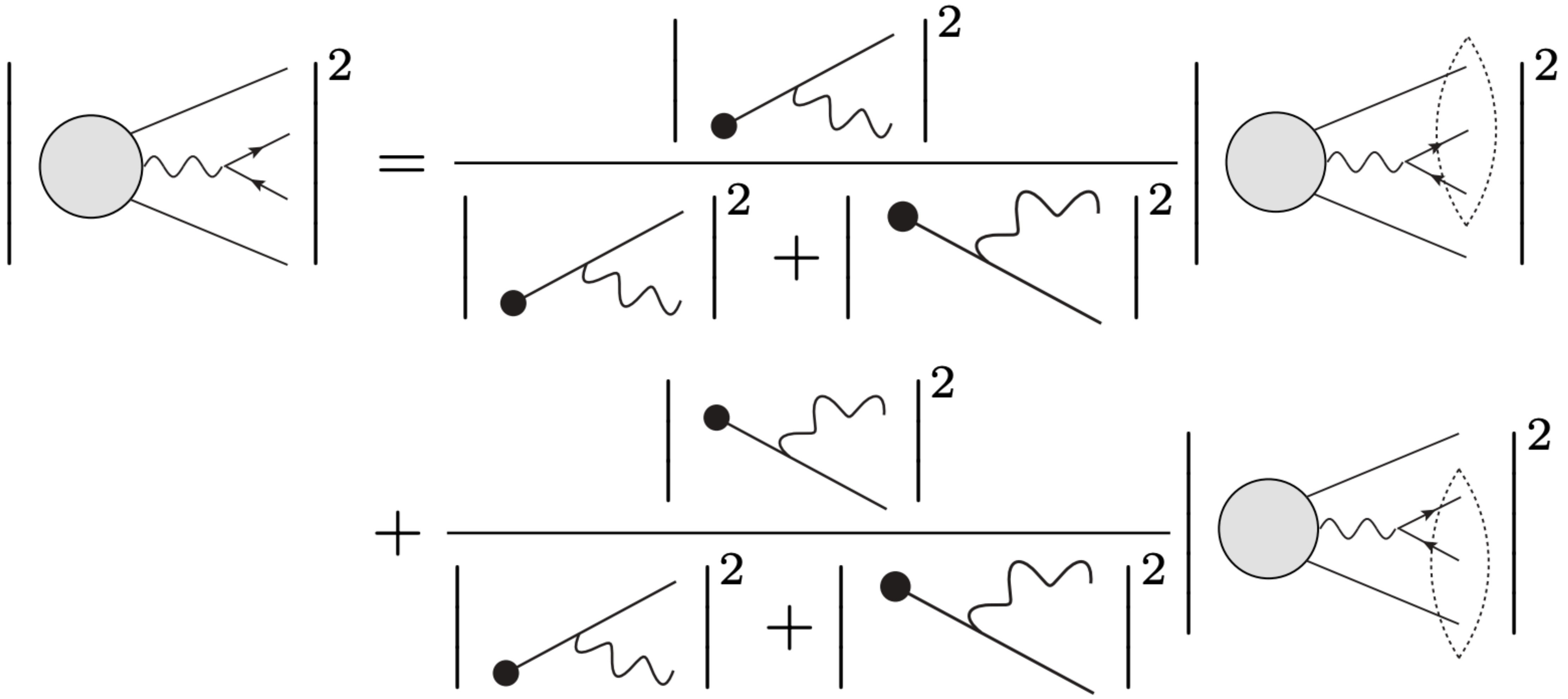
Recoiler Selection

In QCD recoiler determined by colour structure



Gluon splitting: recoiler ambiguous

In EW no such guidance exists



Probabilistic choice to avoid back reaction effects