QED and EW showers in Vincia

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Ronald Kleiss, Peter Skands, Helen Brooks

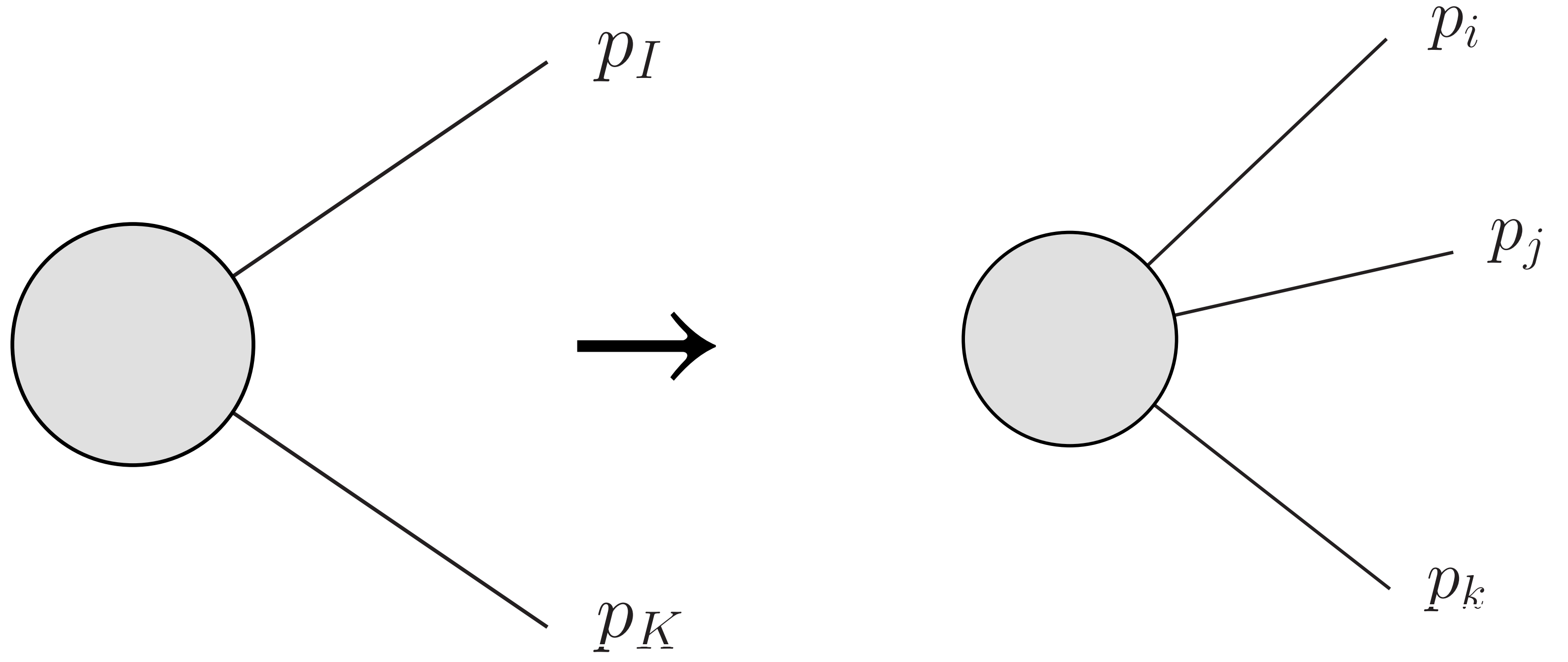
Kleiss, RV 1709.04485
Skands, RV 2002.04939
Kleiss, RV 2002.09248
Brooks, Skands, RV xxxx.xxxxx

One-slide Vincia summary

$$s_{ab} = 2p_a \cdot p_b$$
$$m_{ab}^2 = (p_a + p_b)^2$$

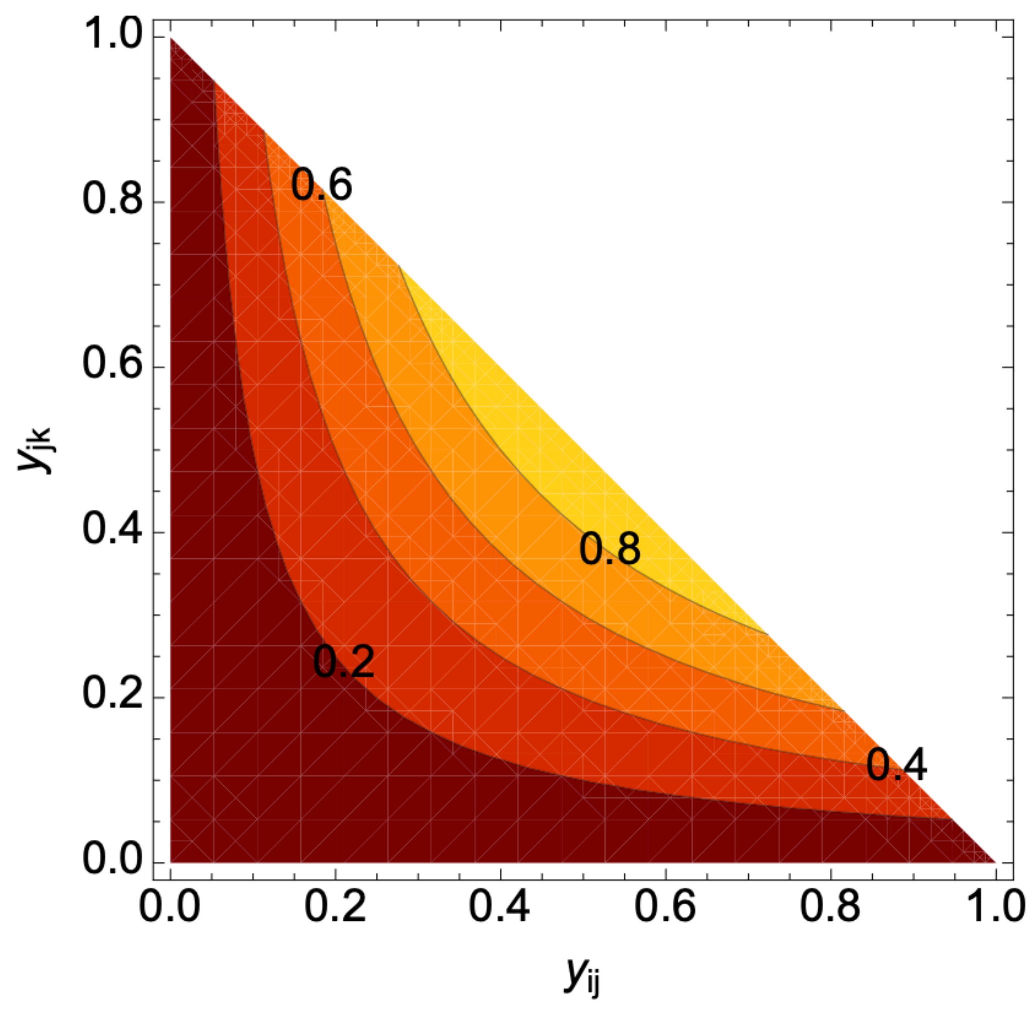
1. Phase space factorisation

$$d\Phi_{\mathrm{ps}} = \frac{1}{16\pi^2}\lambda^{\frac{1}{2}}(m_{IK}^2, m_I^2, m_K^2)\, ds_{ij}\, ds_{jk}\, \frac{d\varphi}{2\pi}$$



2. Ordering scale: Ariadne $p_\perp^2$

$$p_\perp^2 = \frac{s_{ij}s_{jk}}{s_{IK}}$$

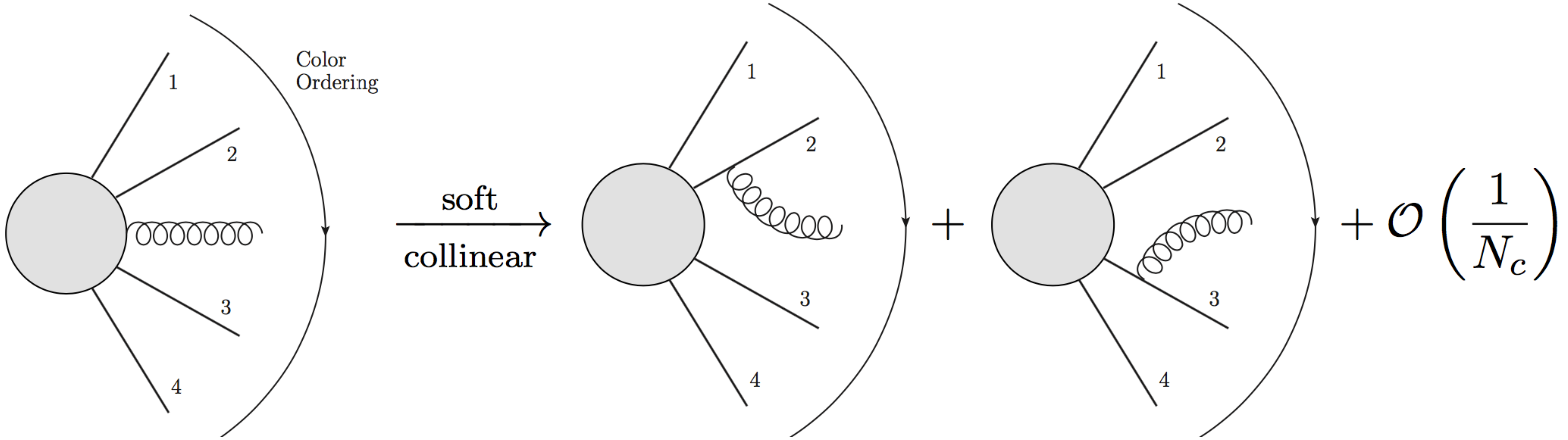


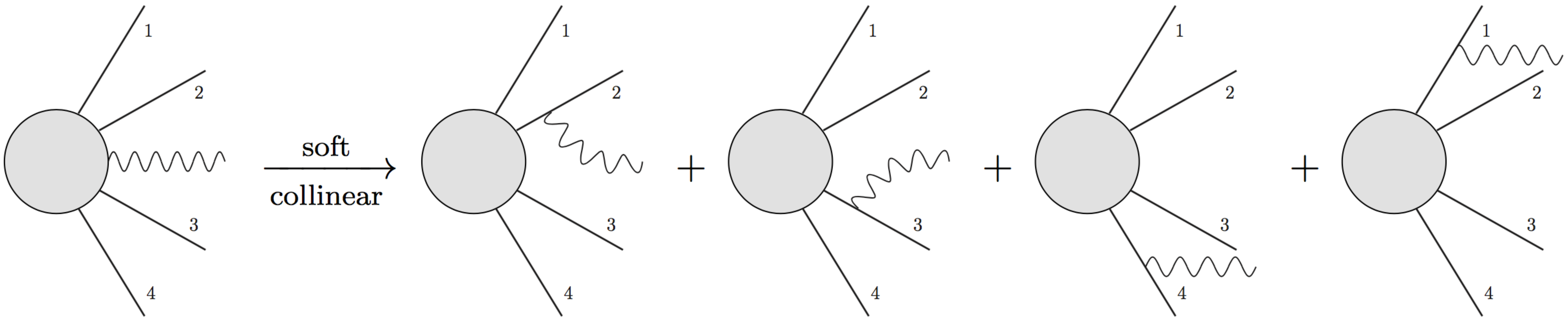
3. Branching kernel: Antenna functions

$$a_{q\bar{q}}(s_{ij}, s_{jk}) = 4\pi\alpha_s C_F \left(2\frac{s_{ik}}{s_{ij}s_{jk}} - 2\frac{m_i^2}{s_{ij}^2} - 2\frac{m_k^2}{s_{jk}^2} + \frac{1}{s_{IK}}\left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}}\right)\right)$$

QED Showers

QCD vs. QED


Color
Ordering
1
2
3
4
soft
collinear
+
$+\mathcal{O}\left(\frac{1}{N_c}\right)$


1
2
3
4
soft
collinear
+

Coherent Photon Radiation

Single branching kernel $\bar{a}^{\mathrm{QED}}(\{p\}, p_j) = -\sum_{\{x,y\}} \sigma_x Q_x \sigma_y Q_y a^{\mathrm{QED}}_{f\bar{f}}(s_{xj}, s_{yj})$

Soft limit

$$|M_{n+1}(\{p\}, p_j)|^2 = -8\pi\alpha \sum_{x,y} \sigma_x Q_x \sigma_y Q_y \frac{s_{xy}}{s_{xj} s_{yj}} |M_n(\{p\}|^2$$

Collinear limit

$$|M_{n+1}(p_1, .., p_i, .., p_n, p_j)|^2 = 4\pi\alpha Q_i^2 \frac{2}{s_{ij}} P_{I\to ij}(z) |M_{n+1}(p_1, .., p_i + p_j, .., p_n)|^2$$

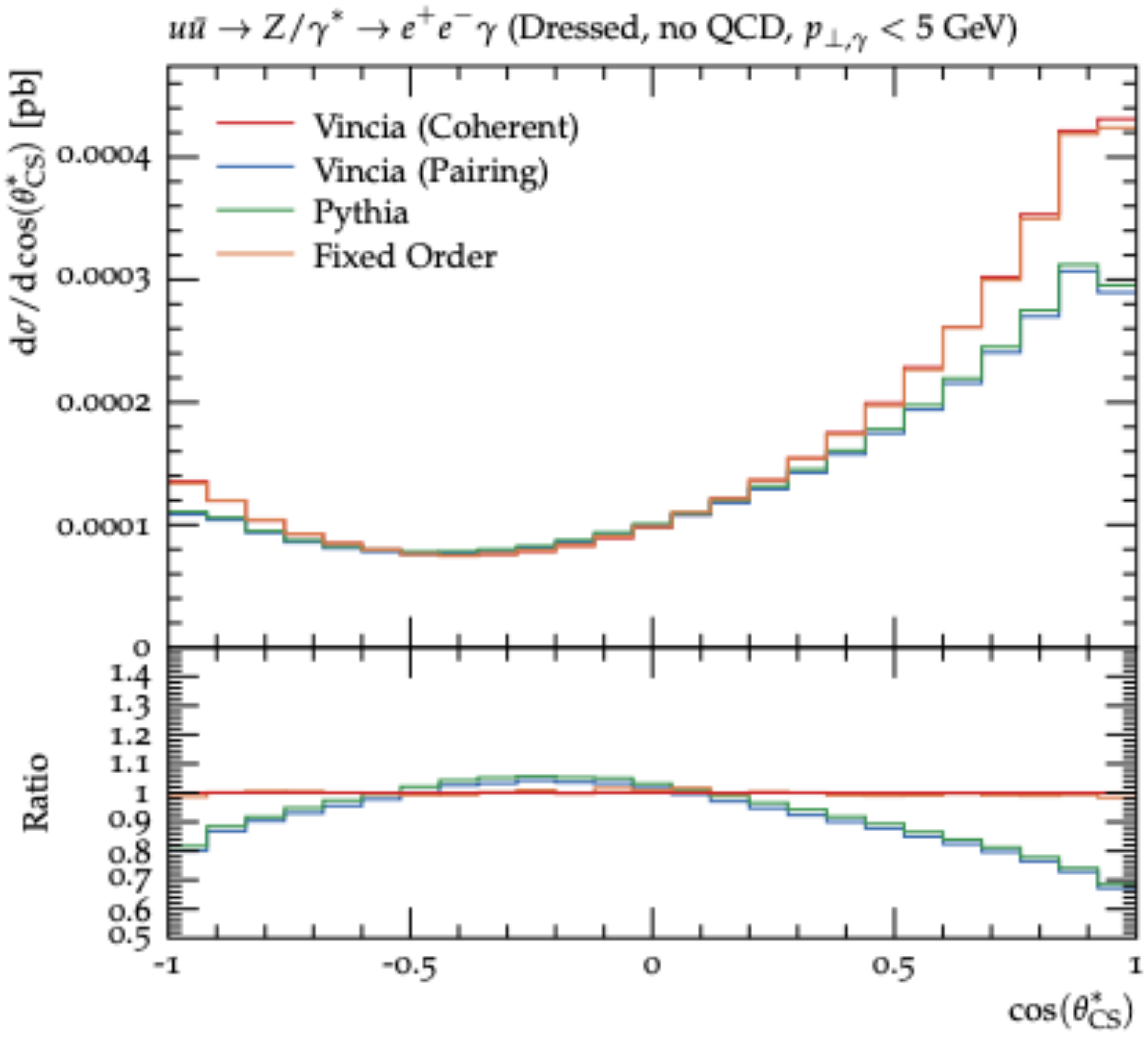
Sectorize the phase space

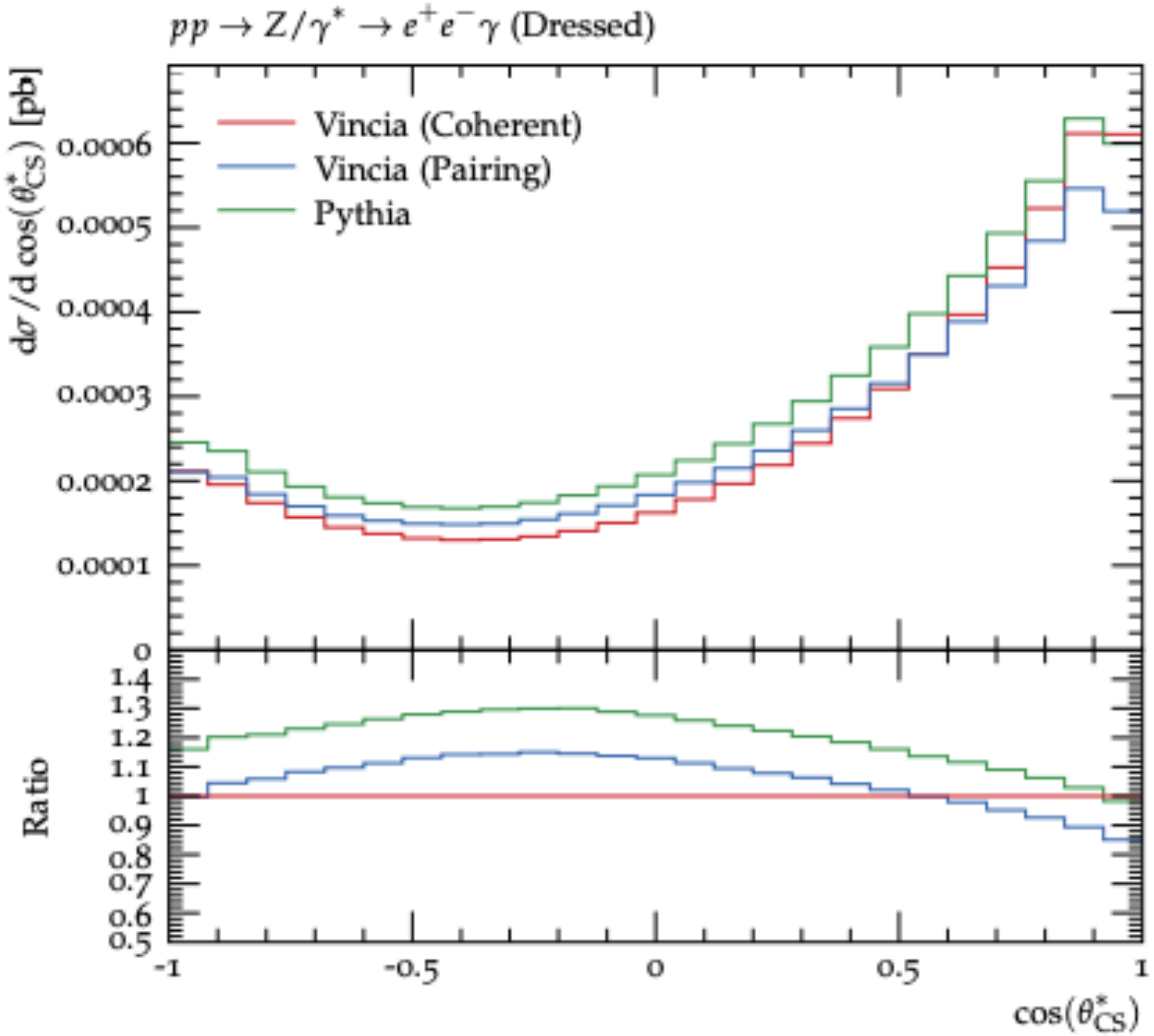
$$|M_{n+1}(\{p\}, p_j)|^2 = \bar{a}^{\mathrm{QED}}(\{p\}, p_j) \sum_{\{x,y\}} \Theta(p^2_{\perp,xy}) |M_n(\{\bar{p}\}_{xy})|^2$$

← x, y does the emission

↑ $p^2_{\perp,xy}$ is the smallest of all $p^2_{\perp}$

High-mass Drell-Yan





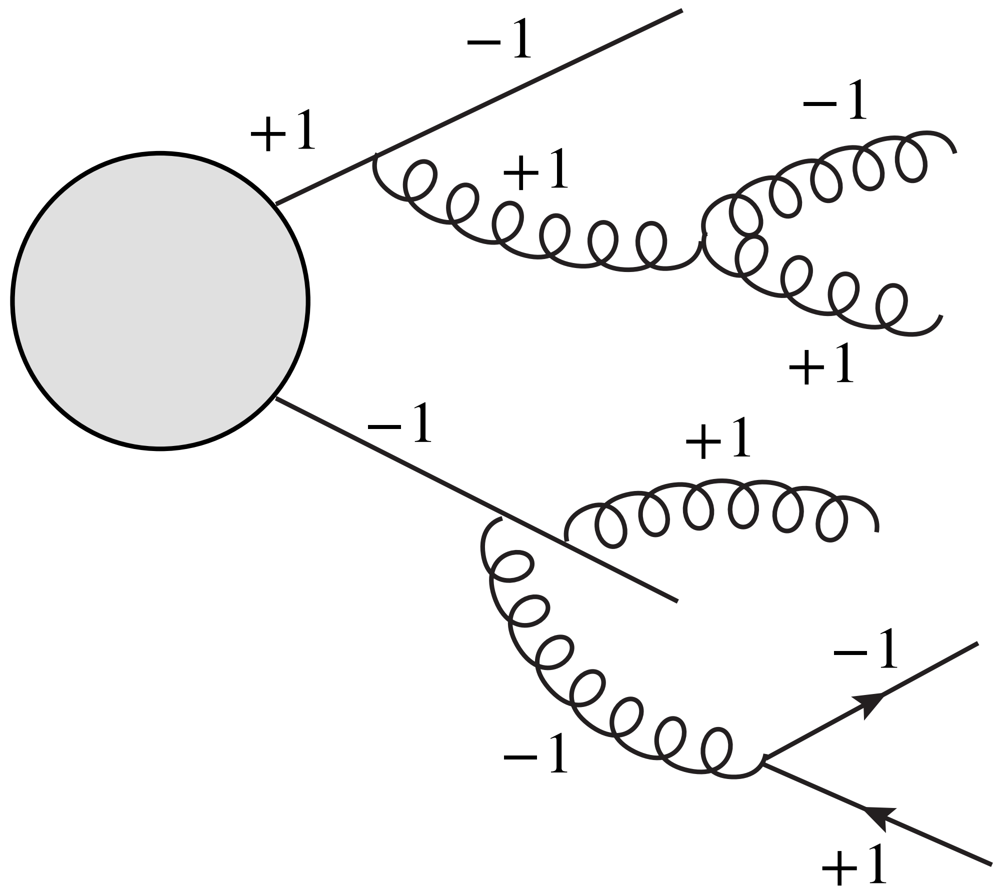
$$\cos\theta^*_{CS} = 2\frac{p^z_{ee}}{|p^z_{ee}|}\frac{p^+_{e^+}p^-_{e^-} - p^-_{e^+}p^+_{e^-}}{m_{ee}\sqrt{m^2_{ee} + p^2_{\perp,ee}}},$$

$m^2_{ee} > 1$ TeV, $p_{\perp,e} > 25$ GeV and $|\eta_e| < 3.5$

$p_{\perp,\gamma} > 0.5$ GeV and $|\eta_\gamma| < 3.5$

Electroweak Showers

EW Showers



- **Real corrections: EW gauge bosons, tops, Higgs part of jets**
- **Virtual corrections: Universal incorporation of Sudakov logs** $\frac{\alpha}{\pi} \ln^2 \left(s/Q^2_{\mathrm{EW}}\right)$

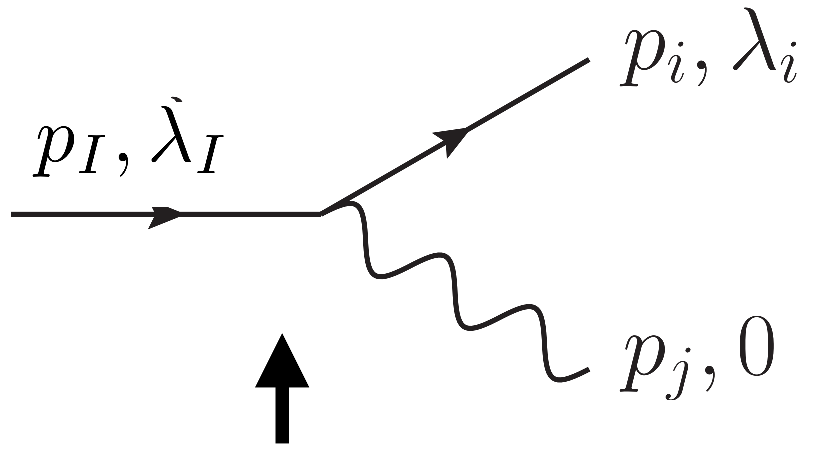
Features of the EW sector

- Chiral ⟶ Helicity showers

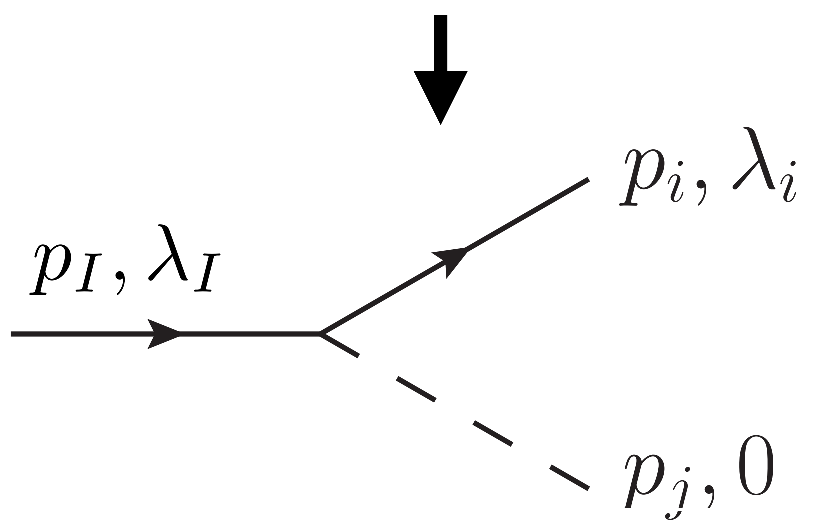
Larkoski, Lopez-Villarejo, Skands 1301.0933

Fischer, Lifson, Stands, 1708.01736

- EW-scale mass corrections
- Longitudinal polarisations / Goldstone bosons
- Neutral boson interference
- Double-counting between QCD and EW
- Resonance-like branchings



$$\epsilon^\mu_0(p) = \frac{1}{m}\left(p^\mu - \frac{m^2}{p \cdot k} k^\mu\right)$$



Lots of Antenna Functions

$$a^{FF}_{f_\lambda \mapsto f_\lambda V_\lambda} = 2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{1}{x_j}$$

$$a^{FF}_{f_\lambda \mapsto f_\lambda V_{-\lambda}} = 2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_i^2}{x_j}$$

$$a^{FF}_{f_\lambda \mapsto f_{-\lambda} V_\lambda} = 2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \left((v-\lambda a) m_i \frac{1}{\sqrt{x_i}} - (v+\lambda a) m_I \sqrt{x_i} \right)^2$$

$$a^{FF}_{f_\lambda \mapsto f_\lambda V_0} = \frac{1}{(m_{ij}^2 - m_I^2)^2} \left[(v-\lambda a) \left(\frac{m_I^2}{m_j} \sqrt{x_i} - \frac{m_i^2}{m_j} \frac{1}{\sqrt{x_i}} - 2 m_j \frac{\sqrt{x_i}}{x_j} \right) + (v+\lambda a) \frac{m_I m_i}{m_j} \frac{x_j}{\sqrt{x_i}} \right]^2$$

$$a^{FF}_{f_\lambda \mapsto f_{-\lambda} V_0} = \frac{(m_I(v+\lambda a) - m_i(v-\lambda a))^2}{m_j^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_j.$$

$$a^{FF}_{V_\lambda \mapsto f_\lambda \bar{f}_{-\lambda}} = 2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_j^2$$

$$a^{FF}_{V_\lambda \mapsto f_{-\lambda} \bar{f}_\lambda} = 2(v+\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_i^2$$

$$a^{FF}_{V_\lambda \mapsto f_{-\lambda} \bar{f}_{-\lambda}} = 2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \left((v+\lambda a) m_i \sqrt{\frac{x_j}{x_i}} + (v-\lambda a) m_j \sqrt{\frac{x_i}{x_j}} \right)^2$$

$$a^{FF}_{V_0 \mapsto f_\lambda \bar{f}_\lambda} = \frac{((v+\lambda a) m_i - (v-\lambda a) m_j)^2}{m_I^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}$$

$$a^{FF}_{V_0 \mapsto f_\lambda \bar{f}_{-\lambda}} = \frac{1}{(m_{ij}^2 - m_I^2)^2} \times \left[(v-\lambda a) \left(2 m_I \sqrt{x_i x_j} - \frac{m_i^2}{m_I} \sqrt{\frac{x_j}{x_i}} - \frac{m_j^2}{m_I} \sqrt{\frac{x_i}{x_j}} \right) + (v+\lambda a) \frac{m_i m_j}{m} \frac{1}{\sqrt{x_i x_j}} \right]^2.$$

$$a^{FF}_{f_\lambda f_\lambda H} = \frac{e^2}{4 s_w^2} \frac{m_i^4}{s_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(\sqrt{x_i} + \frac{1}{\sqrt{x_i}} \right)^2$$

$$a^{FF}_{f_\lambda f_{-\lambda} H} = \frac{e^2}{4 s_w^2} \frac{m_i^2}{s_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_j.$$

$$a^{FF}_{V_\lambda \mapsto V_\lambda H} = \frac{e^2}{s_w^2} \frac{m_v^4}{m_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2}$$

$$a^{FF}_{V_\lambda \mapsto V_0 H} = \frac{e^2}{2 s_w^2} \frac{m_v^2}{m_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_i x_j$$

$$a^{FF}_{V_0 \mapsto V_\lambda H} = \frac{e^2}{2 s_w^2} \frac{m_v^2}{m_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_j}{x_i}$$

$$a^{FF}_{V_0 \mapsto V_0 H} = \frac{e^2}{4 s_w^2} \frac{1}{m_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(m_I^2 - 2 m_i^2 \left(x_i + \frac{1}{x_i} \right) \right)^2.$$

$$a^{FF}_{H \mapsto f_\lambda \bar{f}_\lambda} = \frac{e^2}{4 s_w^2} \frac{m_i^2}{s_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}$$

$$a^{FF}_{H \mapsto f_\lambda \bar{f}_{-\lambda}} = \frac{e^2}{4 s_w^2} \frac{m_i^4}{s_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(\sqrt{\frac{x_i}{x_j}} - \sqrt{\frac{x_j}{x_i}} \right)^2.$$

$$a^{FF}_{V_\lambda \mapsto V_\lambda V_\lambda} = 2 g_v^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{1}{x_i x_j}$$

$$a^{FF}_{V_\lambda \mapsto V_\lambda V_{-\lambda}} = 2 g_v^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_i^3}{x_j}$$

$$a^{FF}_{V_\lambda \mapsto V_{-\lambda} V_\lambda} = 2 g_v^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_j^3}{x_i}$$

$$a^{FF}_{V_\lambda \mapsto V_\lambda V_0} = g_v^2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \frac{(m_I^2 - m_i^2 - \frac{1+x_i}{x_j} m_j^2)^2}{m_j^2}$$

$$a^{FF}_{V_\lambda \mapsto V_0 V_\lambda} = g_v^2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \frac{(m_I^2 - m_j^2 - \frac{1+x_j}{x_i} m_i^2)^2}{m_i^2}$$

$$a^{FF}_{V_\lambda \mapsto V_0 V_0} = \frac{g_v^2}{2} \frac{(m_I^2 - m_i^2 - m_j^2)^2}{m_i^2 m_j^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} x_i x_j.$$

Lots of Antenna Functions (pt. 2)

$$a^{FF}_{V_0\mapsto V_\lambda V_{-\lambda}} = g_v^2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \frac{(m_I^2(1-2x_i) + m_i^2 - m_j^2)^2}{m_I^2}$$
$$a^{FF}_{V_0\mapsto V_\lambda V_0} = \frac{g_v^2}{2} \frac{(m_I^2 - m_i^2 + m_j^2)^2}{m_I^2 m_j^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_j}{x_i}$$
$$a^{FF}_{V_0\mapsto V_0 V_\lambda} = \frac{g_v^2}{2} \frac{(m_I^2 + m_i^2 - m_j^2)^2}{m_I^2 m_i^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_i}{x_j}$$
$$a^{FF}_{V_0\mapsto V_0 V_0} = \frac{g_v^2}{4} \frac{1}{m_I^2 m_i^2 m_j^2} \frac{1}{x_i^2 x_j^2} \times \Big[m_I^4 x_i x_j (x_i - x_j) + 2m_I^2 (m_i^2 x_j^2 (1+x_i) - m_j^2 x_i^2 (1+x_j)) - (m_i^2 - m_j^2)(m_i^2 x_j (1+x_j) + m_j^2 x_i (1+x_i)) \Big]^2.$$

$$a^{II}_{f_\lambda\mapsto f_\lambda V_\lambda} = 2(v-\lambda a)^2 \frac{\tilde{q}_{aj}^2}{(m_A^2 - q_{ai}^2)^2} \frac{1}{x_A} \frac{1}{x_j}$$
$$a^{II}_{f_\lambda\mapsto f_\lambda V_{-\lambda}} = 2(v-\lambda a)^2 \frac{\tilde{q}_{aj}^2}{(m_A^2 - q_{ai}^2)^2} \frac{x_A}{x_j}$$
$$a^{II}_{f_\lambda\mapsto f_{-\lambda} V_\lambda} = 2 \frac{1}{(m_A^2 - q_{ai}^2)^2} \left((v-\lambda a) \frac{m_A}{\sqrt{x_A}} - (v+\lambda a)\sqrt{x_A} m_a \right)^2$$
$$a^{II}_{f_\lambda\mapsto f_\lambda V_0} = \frac{1}{(m_A^2 - q_{ai}^2)^2} \times \left[(v-\lambda a) \left(\frac{m_a^2}{m_j}\sqrt{x_A} - \frac{m_A^2}{m_j}\frac{1}{\sqrt{x_A}} - 2m_j \frac{\sqrt{x_A}}{x_j} \right) + (v+\lambda a) \frac{m_a m_A}{m_j} \frac{x_j}{\sqrt{x_A}} \right]^2$$
$$a^{II}_{f_\lambda\mapsto f_{-\lambda} V_0} = \frac{((v-\lambda a)m_A - (v+\lambda a)m_a)^2}{m_j^2} \frac{\tilde{q}_{aj}^2}{(m_A^2 - q_{ai}^2)^2} \frac{x_j}{x_A}$$

$$a^{FF}_{H\mapsto f_\lambda \bar{f}_\lambda} = \frac{e^2}{4s_w^2} \frac{m_i^2}{s_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}$$
$$a^{FF}_{H\mapsto f_\lambda \bar{f}_{-\lambda}} = \frac{e^2}{4s_w^2} \frac{m_i^4}{s_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(\sqrt{\frac{x_i}{x_j}} - \sqrt{\frac{x_j}{x_i}} \right)^2.$$

$$a^{FF}_{H\mapsto V_\lambda V_{-\lambda}} = \frac{e^2}{s_w^2} \frac{m_v^4}{m_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2}$$
$$a^{FF}_{H\mapsto V_\lambda V_0} = \frac{e^2}{2s_w^2} \frac{m_v^2}{m_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_j}{x_i}$$
$$a^{FF}_{H\mapsto V_0 V_\lambda} = \frac{e^2}{2s_w^2} \frac{m_v^2}{m_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_i}{x_j}$$
$$a^{FF}_{H\mapsto V_0 V_0} = \frac{e^2}{4s_w^2} \frac{1}{m_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(m_I^2 - 2m_v^2 \left(\frac{1}{x_i x_j} - 1 \right) \right)^2.$$

$$a^{II}_{f_\lambda f_\lambda H} = \frac{e^2}{4s_w^2} \frac{m_a^4}{s_w^2} \frac{1}{(m_A^2 - q_{ai}^2)^2} \frac{1}{x_A} \left(\sqrt{x_A} + \frac{1}{\sqrt{x_A}} \right)^2$$
$$a^{II}_{f_\lambda f_{-\lambda} H} = \frac{e^2}{4s_w^2} \frac{m_a^2}{s_w^2} \frac{\tilde{q}_{aj}^2}{(m_A^2 - q_{ai}^2)^2} \frac{1}{x_A} x_j.$$

UCL

Collinear Limits

$$\tilde{m}_{ij}^2 = m_{ij}^2 - \frac{m_i^2}{z^2} - \frac{m_j^2}{(1-z)^2}$$

λ_I	λ_i	λ_j	$f \to f'V$		
λ	λ	λ	$2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{1}{1-z}$	$\to P(z) \propto \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{1+z^2}{1-z}$	Pure gauge
λ	λ	$-\lambda$	$2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{z^2}{1-z}$		
λ	$-\lambda$	λ	$2\frac{1}{(m_{ij}^2-m_I^2)^2}\left(m_I(v-\lambda a)\sqrt{z} - m_i(v+\lambda a)\frac{1}{\sqrt{z}}\right)^2$		Pure gauge
λ	$-\lambda$	$-\lambda$	0	$\to P(z) \propto \frac{m^2}{(m_{ij}^2-m_I^2)^2}$	
λ	λ	0	$\frac{1}{(m_{ij}^2-m_I^2)^2}\left[(v-\lambda a)\left(\frac{m_I^2}{m_j}\sqrt{z} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{z}} - 2m_j\frac{\sqrt{z}}{1-z}\right) + (v+\lambda a)\frac{m_i m_I}{m_j}\frac{1-z}{\sqrt{z}}\right]^2$		Gauge + Goldstone
λ	$-\lambda$	0	$\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}(1-z)\left(\frac{m_i}{m_j}(v-\lambda a) - \frac{m_I}{m_j}(v+\lambda a)\right)^2$	$\to P(z) \propto \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}(1-z)$	Pure Goldstone

Overestimate Determination

$\mathcal{O}(1000)$ types of branchings (all FSR + ffV ISR)

Parameterized overestimate

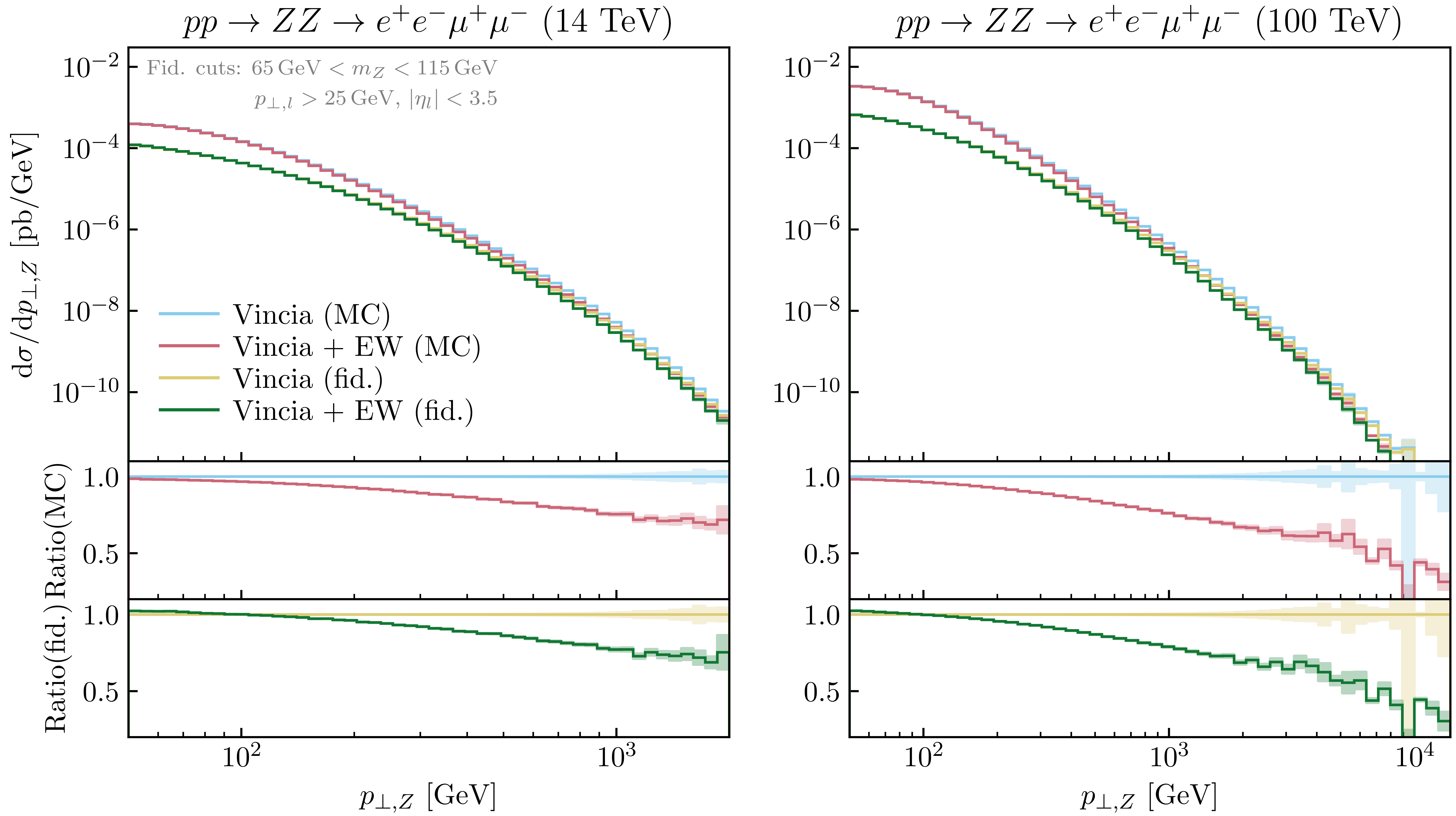
$$a_{\rm trial}^{\rm FF} = \frac{1}{m_{ij}^2 - m_I^2}\left[c_1^{\rm FF} + c_2^{\rm FF}\frac{1}{z} + c_3^{\rm FF}\frac{1}{1-z} + c_4^{\rm FF}\frac{m_I^2}{m_{ij}^2 - m_I^2}\right] \longrightarrow$$ Efficient veto algorithm

For every branching:

- Generate random branchings in random antennae
- Set up *linear programming* system
- Solve numerically

$$\text{Minimize } a_{{\rm trial},i}^{\rm FF} - a_i^{\rm FF}$$
$$\text{While } \forall i:\ a_{{\rm trial},i}^{\rm FF} > a_i^{\rm FF}$$

Virtual Sudakov logs

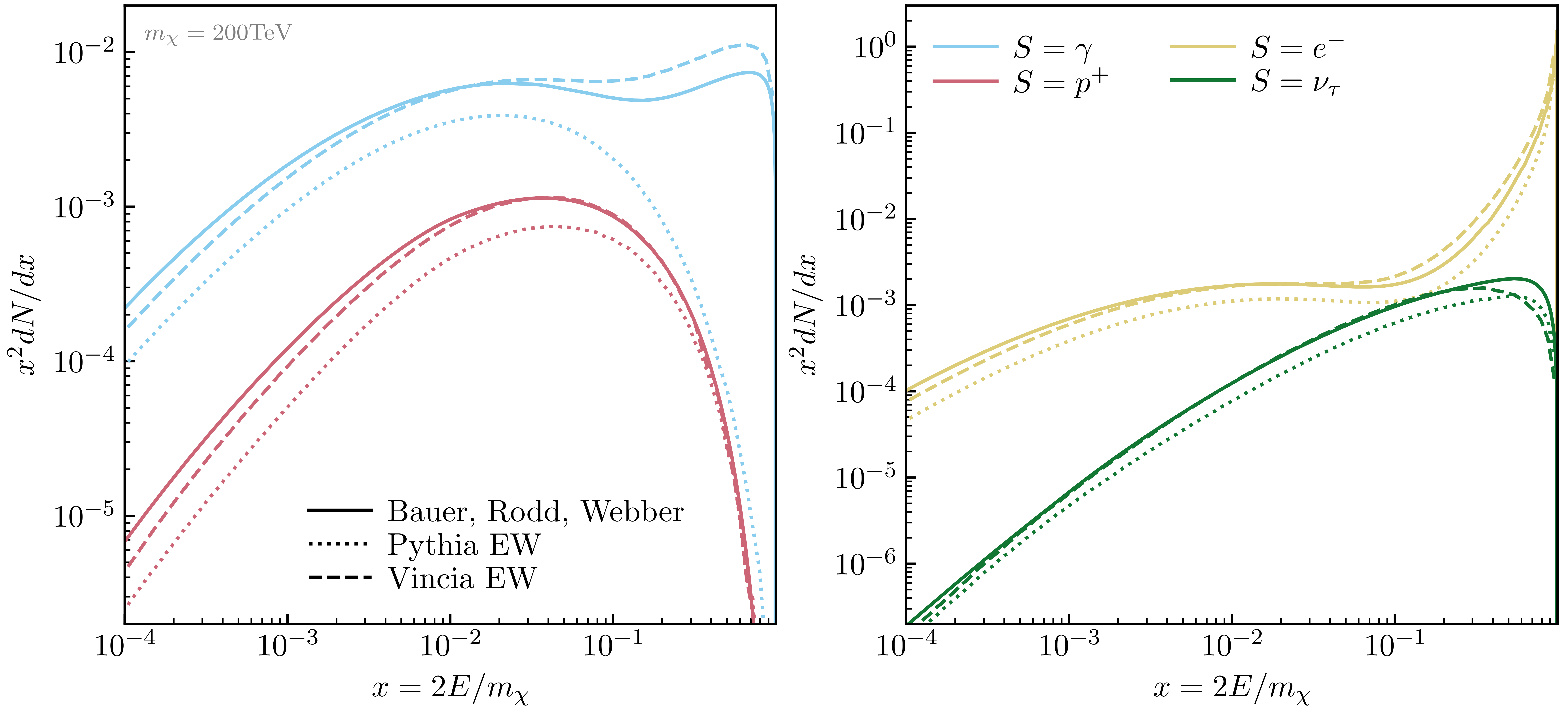


Dark Matter Decay Spectra

Comparison with analytic results

Bauer, Rodd, Webber 2007.15001

$\chi \to \nu_e \bar{\nu}_e \to S$

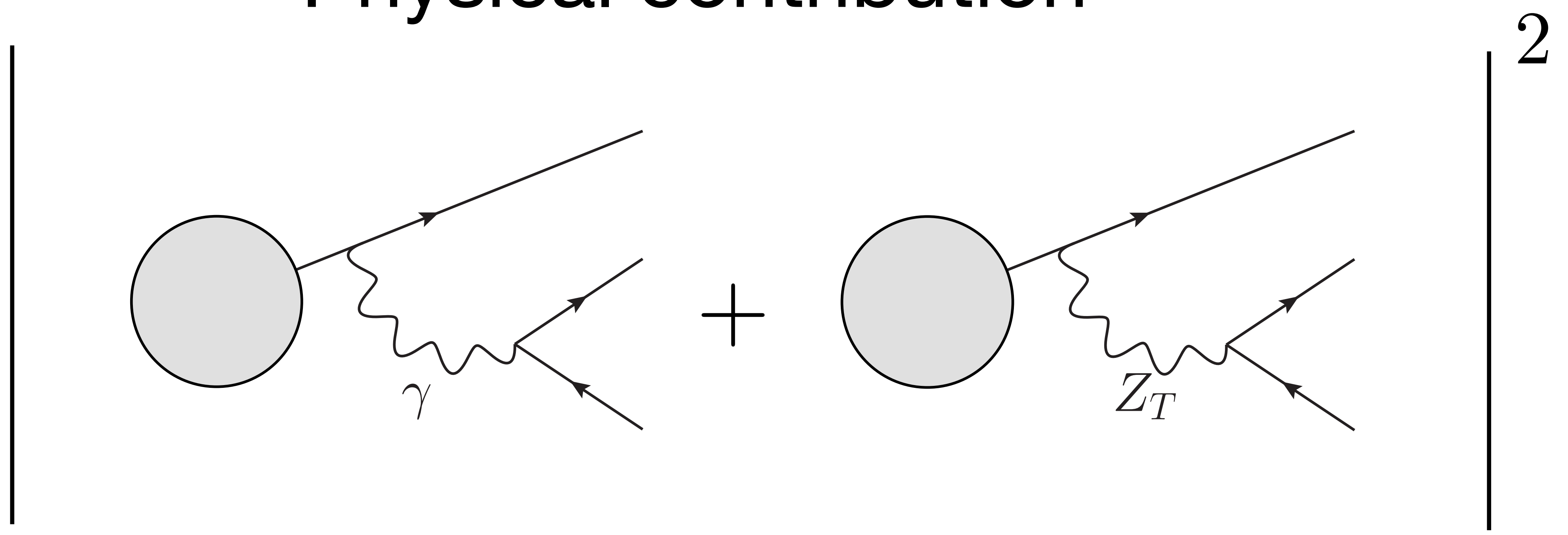


Novel features in the Electroweak Sector

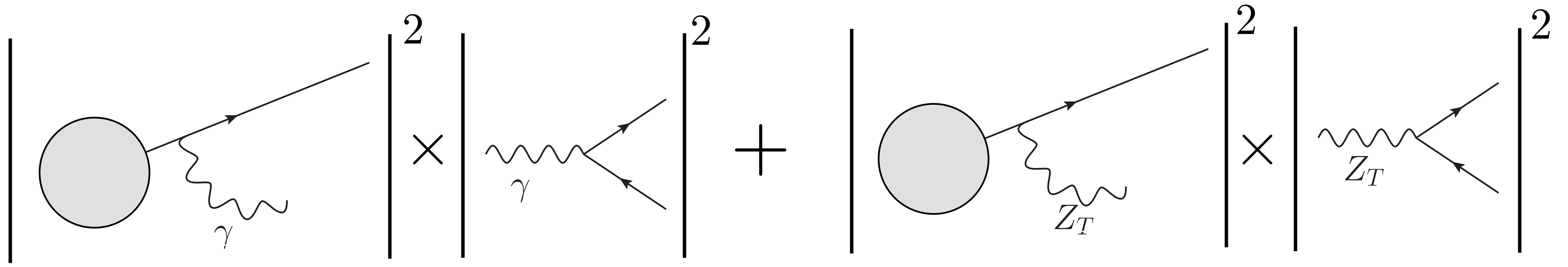
Neutral Boson Interference

Interference between γ, Z_T and h, Z_L

Physical contribution



Shower approximation

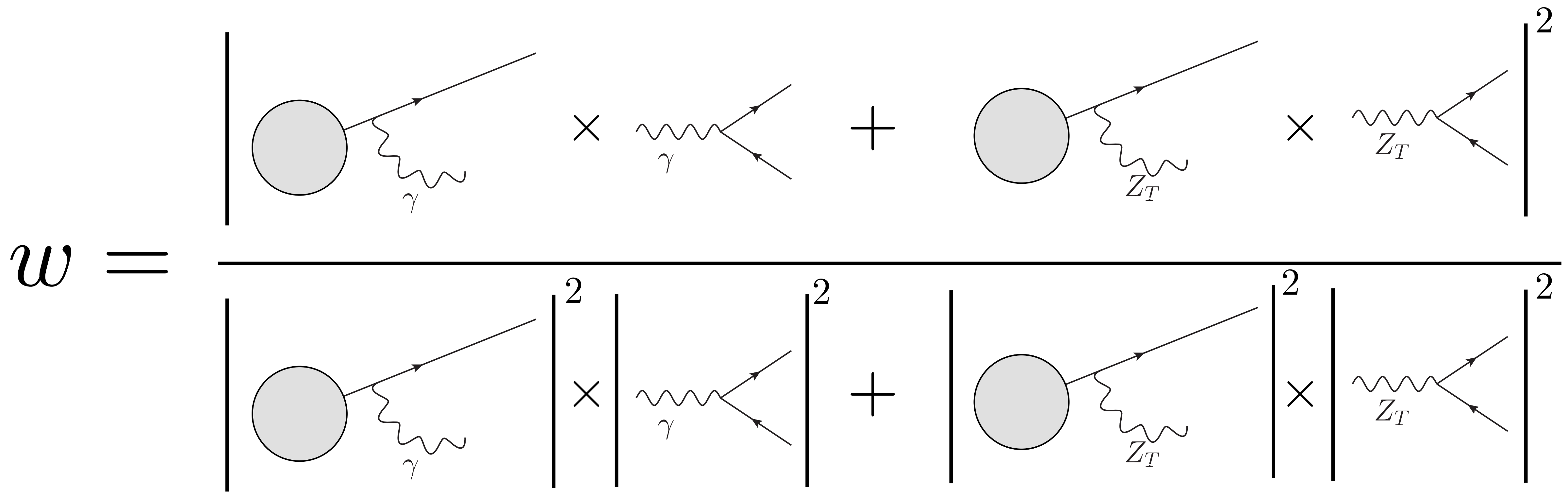


- Complicated solution: Evolve density matrices

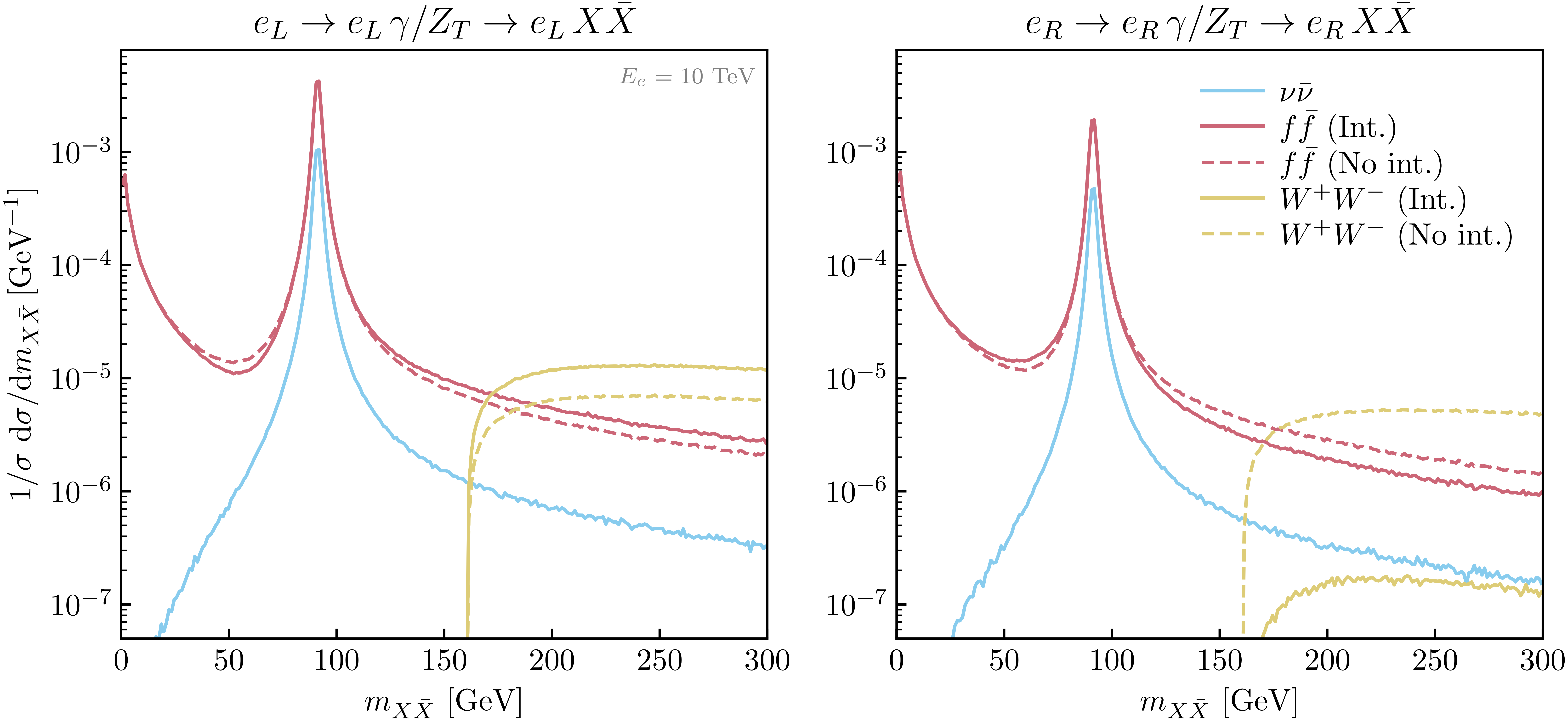
  $\longrightarrow$ Very computationally expensive

- Simple solution: Apply event weight

  $\longrightarrow$ Does not get Sudakov right

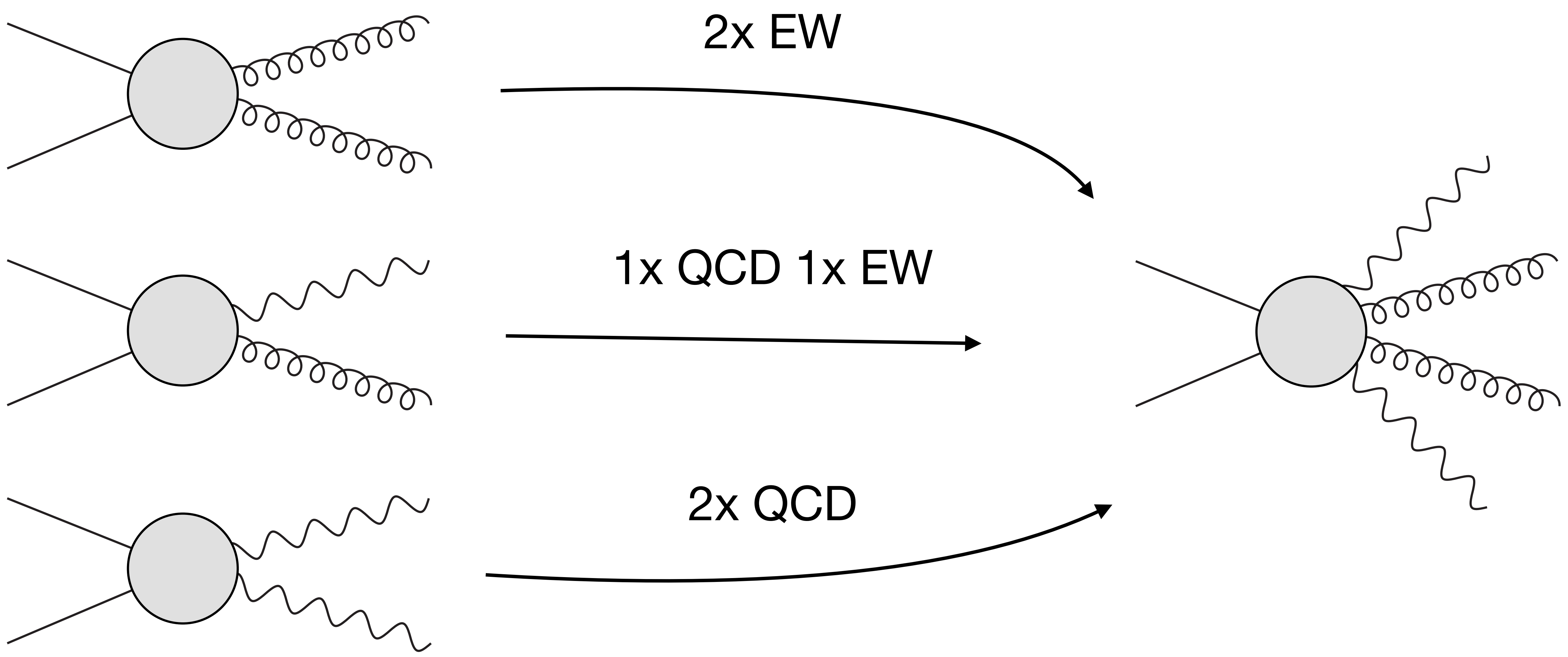


Bosonic Interference

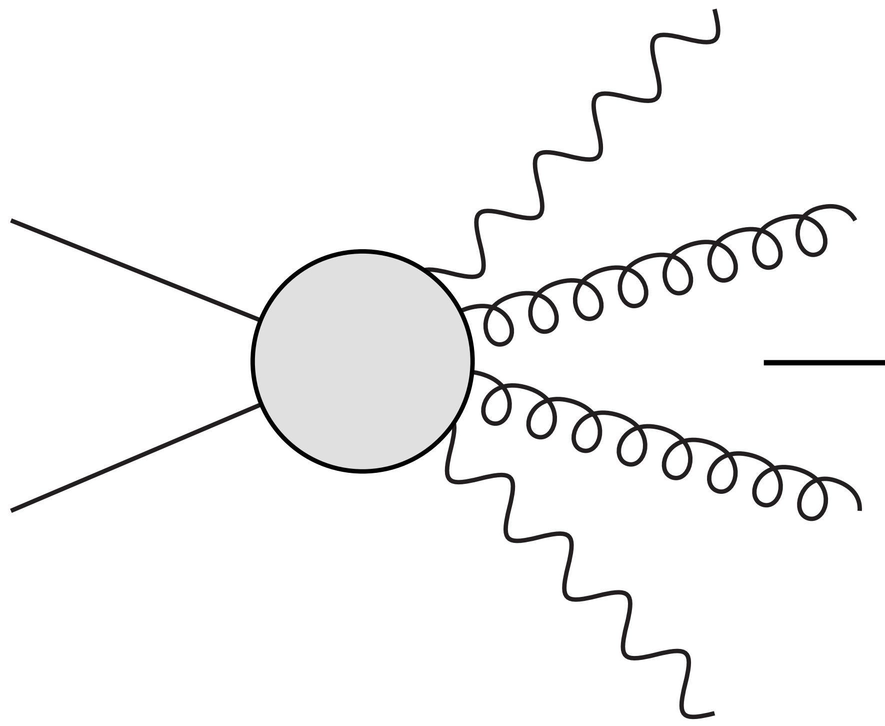


Overlap Veto

Double counting problem



Veto procedure

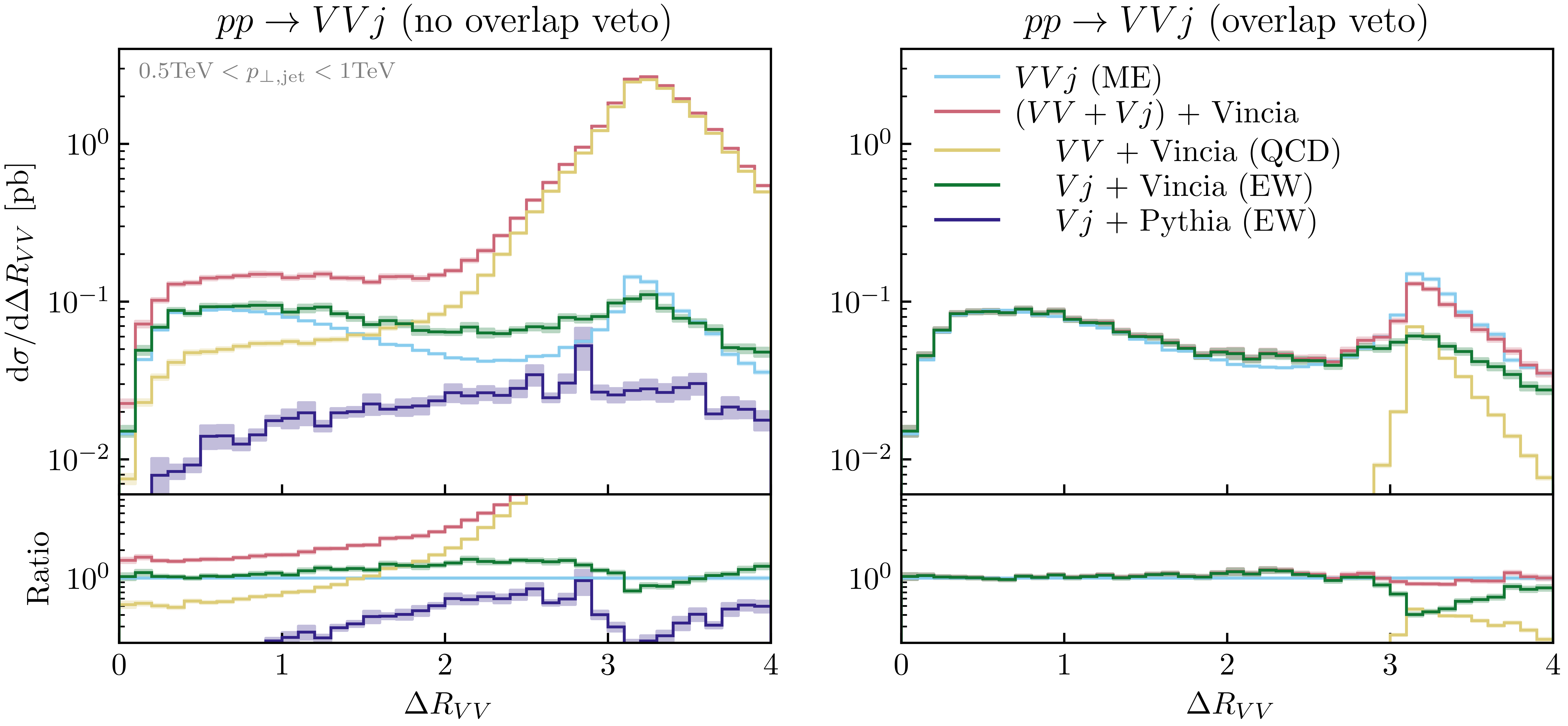


Last emission QCD? $\longrightarrow$ $d_{ij}^{\text{Last}} < \min\left(d_{ij}^{\text{EW}}\right)$ $\longrightarrow$ Accept Branching

Last emission EW? $\longrightarrow$ $d_{ij}^{\text{Last}} < \min\left(d_{ij}^{\text{QCD}}\right)$ $\longrightarrow$ Accept Branching

$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right)\frac{\Delta_{ij}}{R} + m_i^2 + m_j^2 - m^2$$

Overlap Veto



Resonance Matching

Branchings like $t \to bW$, $Z \to q\bar{q}$ etc.

- Large scales:
  EW shower offers best description
- Small scales:
  Breit-Wigner distribution

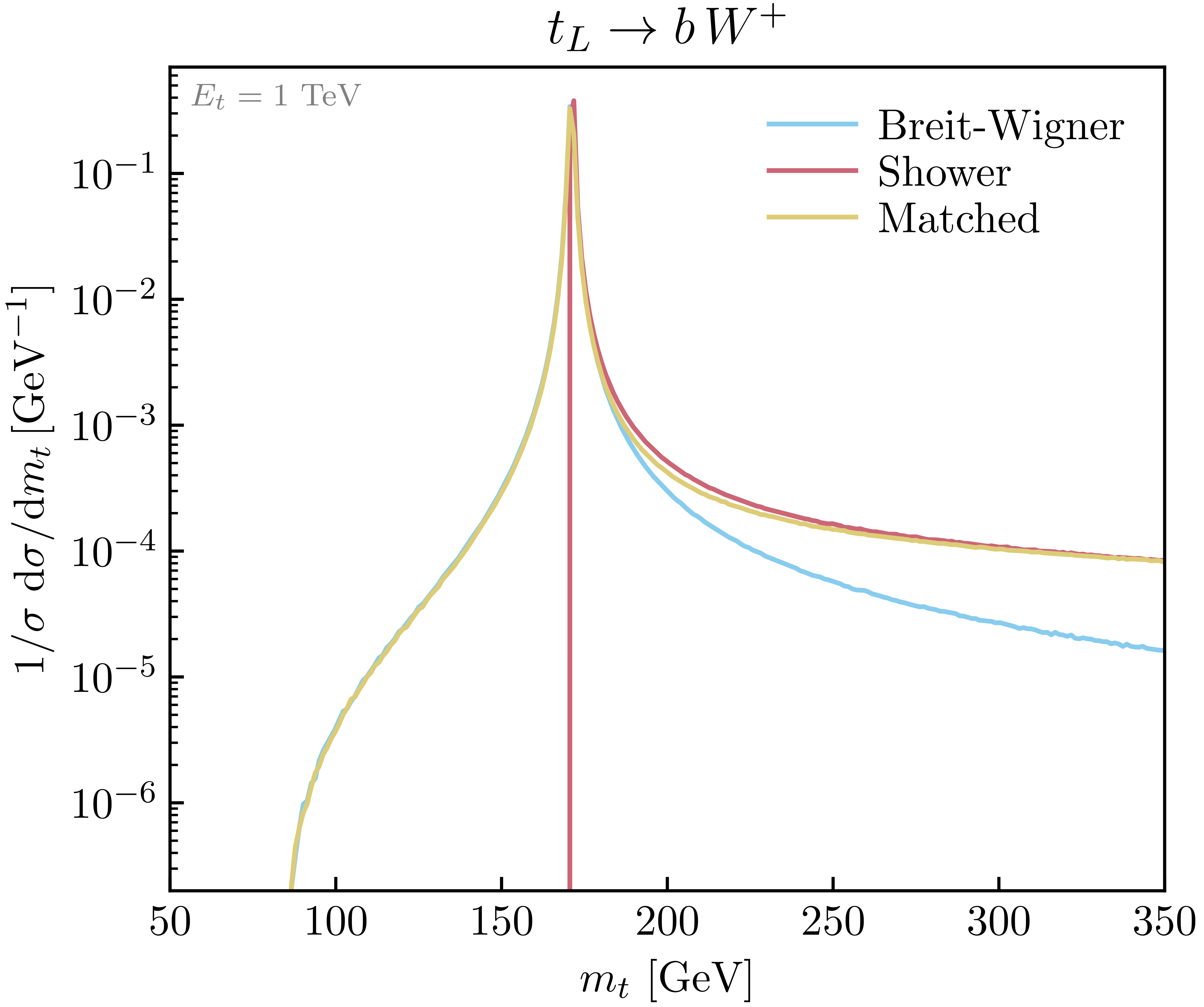
$$\mathrm{BW}(Q^2) \propto \frac{m_0 \Gamma(m)}{Q^4 + m_0^2 \Gamma(m)^2}$$

Matching:

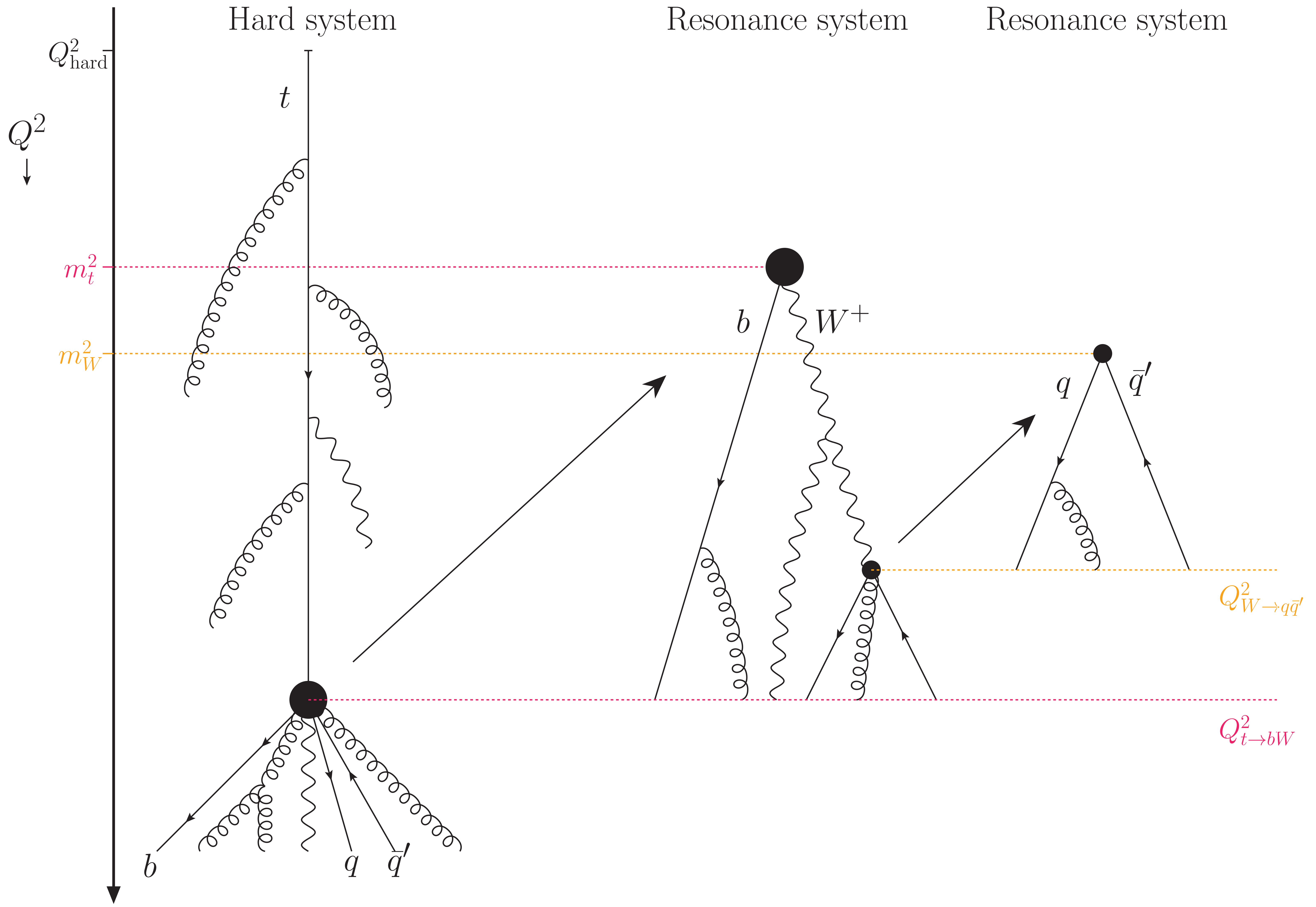
- Sample mass from Breit-Wigner upon production
- Suppress shower by factor

$$\frac{Q^4}{(Q^2 + Q_{\mathrm{EW}}^2)^2}$$

- Decay when shower hits off-shellness scale



Interleaved Resonance Decays



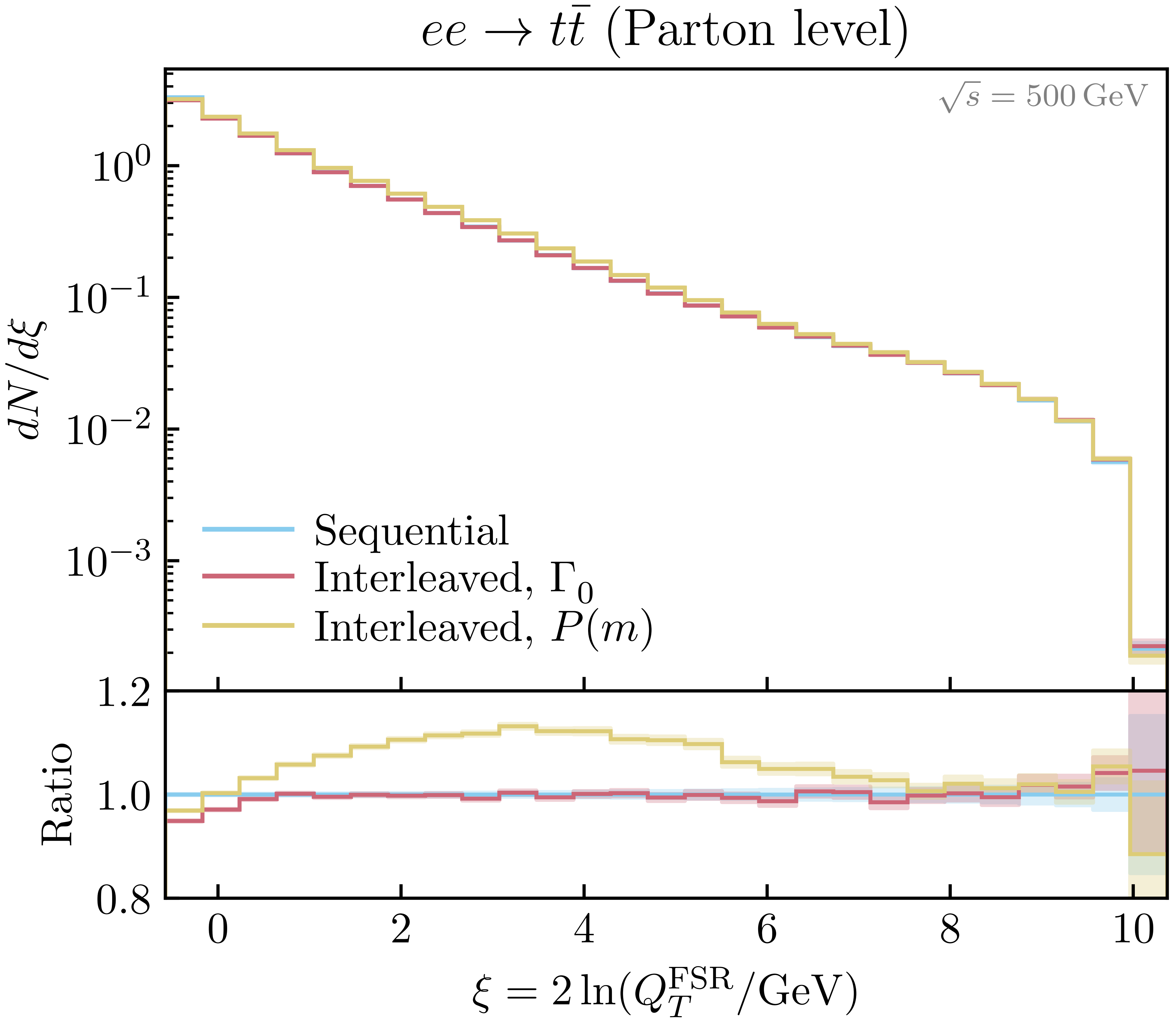
Sequential

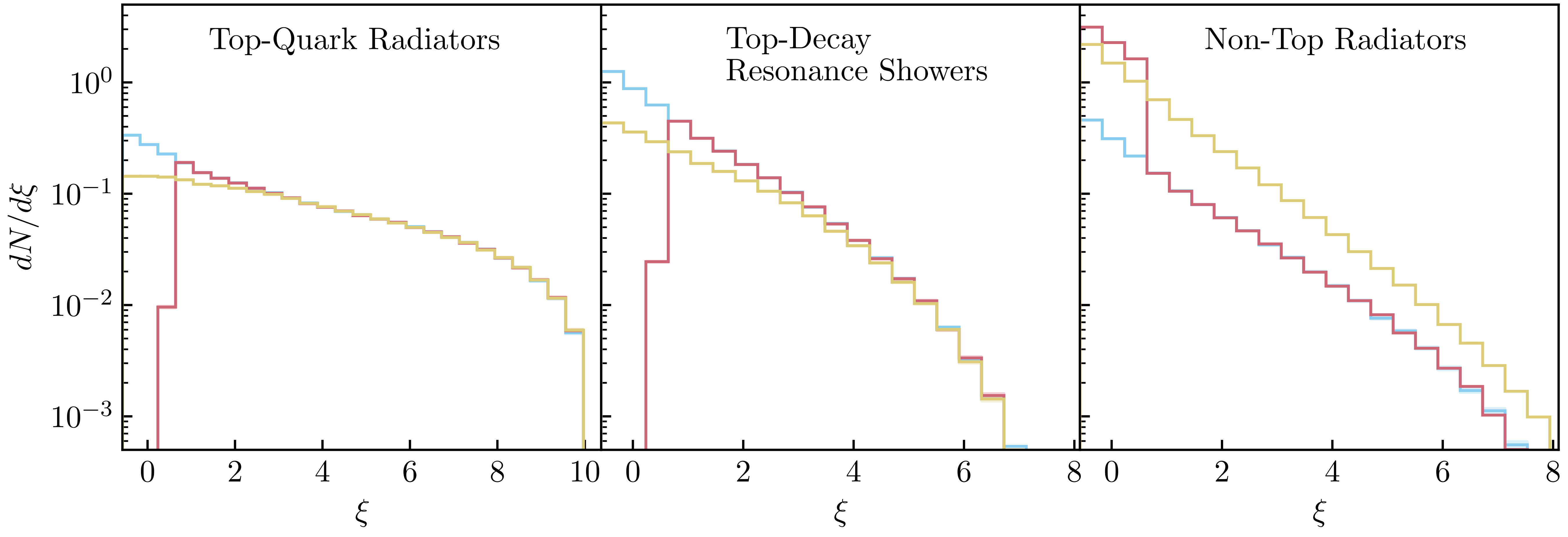
- Complete evolution of the hard system
- Perform resonance shower

Interleaved

- Evolution up to offshellness scale of the resonance
- Perform resonance shower
- Insert showered decay products and continue evolution

Interleaved Resonance Decays





Conclusions

QED Shower

- Includes full soft multipole structure, while interleaved with QCD shower

EW Shower

- Rich physics & many features unique to the EW sector
  - EW symmetry breaking / Goldstone contributions
  - Matching to resonance decays
  - Neutral boson interference
  - Overlap between hard scatterings

- Many other features yet to implement
  - Treatment of soft & spin interference
  - Bloch-Nordsieck violations

Interleaved Resonance Decays

- Physically-intuitive treatment of finite-width / offshell resonances

- More results in

QED & EW shower, and interleaved resonance decays available in Pythia 8.304